

My maths conversation with ChatGPT

When the media discussion about ChatGPT started, I was fascinated by what it seemed to be able to do. Language superior to many chat bots before, eloquent phrasing. It generates ideas, imitates styles, even tells jokes. I was also fascinated by the fact that it does all this based on an algorithm that is quite simple, at the very basis, and that was trained with huge amounts of text. How is it possible that so much of our writing can be replicated, often in high quality, by calculating the probability of the next word based on a specified number of words that came before?

To me this is a major breakthrough, and I am still fascinated. Of course, I wanted to play with this software and make my own experiences. But it was not a high priority until a student asked a weird question about zero divisor in a factor ring of \mathbb{Z} .

The answer was obvious from the material in our lecture, so why were there some students who were not sure that they got it right? It was because ChatGPT gave a different answer.

Now I was intrigued enough to set up an account and start playing.

First in German, for the sake of understanding what happened to my students, and then in English. Now I will share with you what was on my mind, and, of course, the unedited conversation with ChatGPT.

I started with something simple.

Rebecca: Tell me a joke that involves mathematics.

ChatGPT: Why was the math book sad? Because it had too many problems.

Ok, this is a little bit funny. And I wonder whether the programme has given me a joke that it learned during its training or whether it made it up. Also, I realised that, when I asked the question, I was thinking about a joke with mathematicians. Therefore, I asked again.

Rebecca: Tell me a joke that involves mathematicians.

ChatGPT: Why do mathematicians always confuse Halloween and Christmas? Because Oct 31 equals Dec 25!

Ok, this is a little bit funny, but again I consider it a joke about mathematics rather than about mathematicians. But maybe ChatGPT “thinks” that it is funny because of the way that mathematicians think. I tried to find out.

Rebecca: Explain the joke.

ChatGPT: In this joke, the humor comes from a play on words involving the dates of Halloween and Christmas. October 31st is typically celebrated as Halloween, while December 25th is celebrated as Christmas. However, from a mathematical perspective, "Oct" represents the prefix for the number 8 (from the Latin word "octo") and "Dec" represents the prefix for the number 10 (from the Latin word "decem"). So when we write out the dates as numerical values, 10/31 (Oct 31) and 12/25 (Dec 25), they can be seen as equivalent because $8+23$ (Octal 31) is equal to

12+13 (Decimal 25). The joke is that mathematicians, who are used to thinking in terms of numbers and equations, might mix up the two holidays because of this mathematical equivalence.

Frankly, I find that this is a little bit over the top. Yes, we make jokes about binary representations of numbers, of prime numbers, we calculate modulo something and create insider jokes in this way. But I have never seen a joke of the kind that the programme told me. It seems artificial, but I can see why ChatGPT constructs (or cites) such a joke. And the explanation is convincing, too. I wonder whether it can tell a joke that is more about the (presumed) typical personality of mathematicians, or maybe scientists in general. How would I ask questions to get there? First, I got the same jokes again, or very similar jokes. One time, ChatGPT apologises, saying that this joke is not really about maths but at least about numbers. It also tells me the Christmas/Halloween joke again, as an example for a joke that involves mathematical thinking rather than just numbers. But I keep trying, and here is the first new thing that happens:

Rebecca: Tell me a joke with mathematicians.

ChatGPT: Sure, here's a joke involving mathematicians: Three statisticians go hunting for deer. After a while, they spot one in the woods. The first one takes aim and misses by 5 feet to the left. The second one takes aim and misses by 5 feet to the right. The third one jumps up and down and shouts, "We got it! We got it!". In this joke, the third statistician is using a statistical technique called "averaging" to determine the location of the deer. Even though the first two shots missed the deer, the third statistician assumes that the deer is located exactly halfway between the two missed shots, and therefore the third shot must hit it. This assumption is based on the idea that the average of two measurements is a more accurate representation of the true value than either individual measurement on its own.

Aaah, this time it gives me the explanation right away! Ok, I actually like this joke, it is the best one we had so far, and I am ok with the fact that it is about statisticians rather than mathematicians, and that it does not use mathematical thinking for the punchline. I also have to laugh because when children grow up, they often go through a phase where they explain their jokes. Some of my nieces and nephews did this, and I find it hilarious. Now I want to know what ChatGPT "thinks" about its own mathematical capabilities. At some point I have to refresh my browser and it makes me laugh that I get asked to confirm that I am human. Also, the programme warns me that it might "occasionally generate false information".

Rebecca: How confident are you with mathematics?

ChatGPT: As an AI language model, I have been trained on a vast amount of mathematical concepts and formulas, and I can perform various mathematical operations with a high degree of accuracy. However, I am not infallible and may occasionally make mistakes. My confidence in mathematics is based on the quality and accuracy of the data and algorithms used to train me, and my ability to apply that knowledge to new problems.

Ok, let us see!

Rebecca: What is a square?

ChatGPT: A square is a geometric shape that has four equal sides and four equal angles, each measuring 90 degrees. It is a special type of rectangle, where all four sides are equal in length, and all four corners are right angles. Squares are two-dimensional objects that lie in a plane, and they are often represented using a square symbol, such as " \square ".

Squares are a common shape in geometry and mathematics, and they have several unique properties. For example, the perimeter of a square is equal to the sum of its four sides, and the area of a square is equal to the product of its two adjacent sides. Additionally, the diagonal of a square is equal to the square root of two times the length of its sides. Squares are commonly used in many applications, such as in tile work, graphic design, and architecture. They are also used in various mathematical concepts, such as the Pythagorean theorem and trigonometry.

This seems ok to me, and I begin to notice that I associate a voice to the answers, a tone, I begin to anthropomorphise the programme. Moreover, the answer contains information I did not ask for, embellishments, and I wonder why this happens. Why did the programme not just give a short, precise definition? Maybe I have to give some context and explain that I want to have a proper mathematical conversation.

Rebecca: How confident are you about Linear Algebra?

ChatGPT: As an AI language model, I am proficient in linear algebra and can perform a variety of operations, such as matrix multiplication, calculating determinants, solving systems of linear equations, finding eigenvectors and eigenvalues, and performing various transformations. However, it is important to note that my proficiency is based on the algorithms and methods programmed into my software, and I don't have a human-like intuition or understanding of the concepts beyond what I have been trained on. Therefore, my confidence level can be considered high in performing mathematical operations related to linear algebra, but it is not the same as that of a human expert who possesses a deep understanding of the subject.

Rebecca: Please give the mathematical definition of a vector space.

A vector space is a collection of objects, called vectors, which can be added together and multiplied by scalars (numbers), satisfying the following properties:

1. Closure under vector addition: For any two vectors u and v in the vector space, their sum $u + v$ is also in the vector space.
2. Commutativity of vector addition: For any two vectors u and v in the vector space, $u + v = v + u$.
3. Associativity of vector addition: For any three vectors u , v , and w in the vector space, $(u + v) + w = u + (v + w)$.
4. Existence of a zero vector: There exists a vector 0 such that for any vector u in the vector space, $u + 0 = u$.
5. Existence of additive inverse: For any vector u in the vector space, there exists a vector $-u$ in the vector space such that $u + (-u) = 0$.
6. Closure under scalar multiplication: For any scalar a and any vector u in the vector space, the product $a*u$ is also in the vector space.
7. Distributivity of scalar multiplication over vector addition: For any scalar a and any vectors u and v in the vector space, $a*(u + v) = a*u + a*v$.
8. Distributivity of scalar multiplication over scalar addition: For any scalars a and b and any vector u in the vector space, $(a+b)*u = a*u + b*u$.
9. Associativity of scalar multiplication: For any scalars a and b and any vector u in the vector space, $(ab)*u = a*(b*u)$.
10. Identity element of scalar multiplication: For any vector u in the vector space, $1*u = u$, where 1 is the multiplicative identity of the scalar field.

These properties are also known as the axioms of a vector space.

Rebecca: Where do the scalars come from?

ChatGPT: The scalars in the definition of a vector space come from a field. A field is a set of numbers with two operations, typically denoted as addition and multiplication, that satisfy certain axioms. The most commonly used fields in mathematics are the real numbers, denoted by \mathbb{R} , and the complex numbers, denoted by \mathbb{C} . In the definition of a vector space, the scalars are used to multiply vectors, as well as to add and subtract vectors. For example, if we have a vector u and a scalar a , we can form the product $a*u$, which is a new vector in the vector space. Similarly, if we have two vectors u and v , we can add them together to obtain a new vector $u+v$, where both u and v are multiplied by the scalar 1 before being added. The scalars allow us to manipulate and transform vectors in a meaningful way. It is important to note that the choice of field can have a significant impact on the properties and behavior of a vector space. For example, a vector space over the field of real numbers may have different properties than a vector space over the field of complex numbers.

Rebecca: What does it mean for a vector space to be finite-dimensional?

ChatGPT: A vector space is said to be finite-dimensional if it has a finite basis. A basis of a vector space is a set of vectors that can be used to express any other vector in the space as a linear combination of the basis vectors. In other words, a basis is a set of linearly independent vectors that span the vector space. If a vector space has a finite basis, then it is said to be finite-dimensional. The dimension of a finite-dimensional vector space is defined as the number of vectors in any basis for that space. For example, the vector space \mathbb{R}^2 (the set of all ordered pairs of real numbers) is finite-dimensional with dimension 2, because it has a basis consisting of two linearly independent vectors, namely $(1,0)$ and $(0,1)$. Any vector in \mathbb{R}^2 can be expressed as a linear combination of these two basis vectors. In contrast, the vector space of all polynomials with real coefficients is infinite-dimensional, because it does not have a finite basis. Any finite set of polynomials will fail to span the entire space, so we need an infinite number of linearly independent polynomials to form a basis for the space.

The programme is so chatty! But I have several questions now, some statements seem ambiguous to me. So I keep asking.

Rebecca: In your sentence beginning with "In other words", you seem to claim that every generating set is a basis. Please be more precise.

ChatGPT: I apologize for any confusion caused by my previous response. You are correct that not every generating set is a basis. Let me clarify my statement:

A basis is a set of vectors that is linearly independent and spans the vector space. This means that every vector in the space can be expressed as a unique linear combination of the basis vectors. A generating set, on the other hand, is a set of vectors that spans the vector space. However, a generating set may not be linearly independent, which means that some of its vectors can be expressed as linear combinations of the others. So, to be more precise, not every generating set is a basis, but every basis is a generating set. A generating set can be turned into a basis by removing linearly dependent vectors and adding linearly independent vectors, if necessary, until the resulting set is linearly independent and spans the vector space.

Rebecca: What is a linearly dependent vector?

ChatGPT: A linearly dependent vector is a vector that can be expressed as a linear combination of other vectors in a given set. More formally, given a set of vectors $\{v_1, v_2, \dots, v_n\}$, a vector v_i is said to be linearly dependent if it can be written as a linear combination of the other vectors in the set:

$$v_i = a_1 v_1 + a_2 v_2 + \dots + a_{i-1} v_{i-1} + a_{i+1} v_{i+1} + \dots + a_n v_n$$

where $a_1, a_2, \dots, a_{i-1}, a_{i+1}, \dots, a_n$ are scalars (not all zero).

In other words, a vector is linearly dependent if it does not add any new information to the set of vectors, because it can be expressed as a linear combination of the others. If all vectors in a set are linearly independent, then the set is said to be linearly independent. Otherwise, the set is said to be linearly dependent. It is important to note that in a linearly dependent set, at least one vector can be expressed as a linear combination of the others. This means that the set contains redundant information and can be reduced in size without losing any information. In contrast, a linearly independent set contains no redundant information and is the smallest possible set that can span the same subspace.

Again, very chatty. Lots of things I did not ask for. Let us go into something more subtle.

Rebecca: Define the polynomial ring over the field of real numbers.

ChatGPT: The polynomial ring over the field of real numbers, denoted by $R[x]$, is the set of all polynomials in the variable x with coefficients from the real numbers. A polynomial in $R[x]$ has the form:

$$p(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$$

where $a_0, a_1, a_2, \dots, a_n$ are real numbers and n is a non-negative integer. The symbol x is called the indeterminate or variable. The addition and multiplication of polynomials in $R[x]$ are defined in the usual way. Specifically, given two polynomials $p(x)$ and $q(x)$ in $R[x]$, we define:

- The sum of $p(x)$ and $q(x)$ as the polynomial $r(x) = p(x) + q(x)$, where the coefficients of $r(x)$ are obtained by adding the corresponding coefficients of $p(x)$ and $q(x)$.
- The product of $p(x)$ and $q(x)$ as the polynomial $r(x) = p(x)q(x)$, where the coefficients of $r(x)$ are obtained by multiplying the corresponding coefficients of $p(x)$ and $q(x)$, and adding the results for each power of x .

The polynomial ring $R[x]$ is a commutative ring with identity, meaning that addition and multiplication are both commutative, associative, and distributive over each other, and that there exist additive and multiplicative identities. The polynomial ring $R[x]$ is an important object of study in algebra and analysis, with applications in many areas of mathematics, science, and engineering.

I notice that I get annoyed by all these „this is an important ...” statements that I did not ask for. Why is the programme extending the answers in this unnecessary way?

Also, the answer is not satisfactory because it is a description rather than a definition. What is x ? What is a variable? Let us ask.

Rebecca: Sketch a proof for why such a polynomial ring exists.

ChatGPT: To show that the polynomial ring $R[x]$ over the field of real numbers R exists, we need to verify that it satisfies the axioms of a commutative ring with identity.

First, we define the set $R[x]$ as the set of all formal expressions of the form:

$$p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

where $a_0, a_1, a_2, \dots, a_n$ are real numbers and n is a non-negative integer. These formal expressions are called polynomials with real coefficients, and the symbol x is a formal variable. We define addition and multiplication of polynomials in the usual way. Specifically, given two polynomials $p(x)$ and $q(x)$ in $R[x]$, we define:

- The sum of $p(x)$ and $q(x)$ as the polynomial $r(x) = p(x) + q(x)$, where the coefficients of $r(x)$ are obtained by adding the corresponding coefficients of $p(x)$ and $q(x)$.
- The product of $p(x)$ and $q(x)$ as the polynomial $r(x) = p(x)q(x)$, where the coefficients of $r(x)$ are obtained by multiplying the corresponding coefficients of $p(x)$ and $q(x)$, and adding the results for each power of x .

It can be shown that addition and multiplication of polynomials satisfy the axioms of a commutative ring with identity, which are:

1. Addition and multiplication are commutative: $p(x) + q(x) = q(x) + p(x)$ and $p(x)q(x) = q(x)p(x)$ for any polynomials $p(x)$ and $q(x)$ in $R[x]$.
2. Addition and multiplication are associative: $(p(x) + q(x)) + r(x) = p(x) + (q(x) + r(x))$ and $(p(x)q(x))r(x) = p(x)(q(x)r(x))$ for any polynomials $p(x)$, $q(x)$, and $r(x)$ in $R[x]$.
3. Addition and multiplication are distributive over each other: $p(x)(q(x) + r(x)) = p(x)q(x) + p(x)r(x)$ and $(p(x) + q(x))r(x) = p(x)r(x) + q(x)r(x)$ for any polynomials $p(x)$, $q(x)$, and $r(x)$ in $R[x]$.
4. There exist additive and multiplicative identities: there exist polynomials 0 and 1 in $R[x]$ such that $p(x) + 0 = p(x)$ and $p(x)1 = p(x)$ for any polynomial $p(x)$ in $R[x]$.

Therefore, the polynomial ring $R[x]$ over the field of real numbers R exists and is a commutative ring with identity.

Hm. Did the programme just tell me that this structure exists because we can write symbols down? Is this all?

Rebecca: This is not a proof. Please try again.

ChatGPT: I apologize for the unclear response earlier. Here is a more detailed proof that a polynomial ring over the field of real numbers exists: Let R be the field of real numbers. We define the set of all polynomials with coefficients in R as the set of all formal expressions of the form:

$$p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

where $a_0, a_1, a_2, \dots, a_n$ are real numbers and n is a non-negative integer. These formal expressions are called polynomials with real coefficients, and the symbol x is a formal variable. We can define two operations on the set of polynomials:

1. Addition: Given two polynomials $p(x)$ and $q(x)$ in $R[x]$, we define their sum as $r(x) = p(x) + q(x)$, where the coefficients of $r(x)$ are obtained by adding the corresponding coefficients of $p(x)$ and $q(x)$.
2. Multiplication: Given two polynomials $p(x)$ and $q(x)$ in $R[x]$, we define their product as $r(x) = p(x)q(x)$, where the coefficients of $r(x)$ are obtained by multiplying the corresponding coefficients of $p(x)$ and $q(x)$, and adding the results for each power of x .

We can show that these operations satisfy the following axioms of a commutative ring with identity:

1. $R[x]$ is an abelian group under addition.
 - Addition is associative, commutative, and has the identity element 0, which is the polynomial with all coefficients equal to zero.
 - Each polynomial has an additive inverse, which is obtained by negating all the coefficients.
2. $R[x]$ is a monoid under multiplication.
 - Multiplication is associative and has the identity element 1, which is the polynomial with $a_0=1$ and all other coefficients equal to zero.
3. Multiplication distributes over addition.
 - Multiplication distributes over addition in $R[x]$.
4. Multiplication is commutative.
 - Multiplication is commutative in $R[x]$.

Therefore, the set of polynomials with coefficients in R , together with the operations of addition and multiplication defined above, forms a commutative ring with identity, which we denote by $R[x]$. This proves the existence of the polynomial ring over the field of real numbers.

Rebecca: What is x ? What is a formal expression?

ChatGPT: In the context of a polynomial ring over the field of real numbers, x is a formal variable. It is used as a placeholder for the values that the polynomial may take. A formal expression is a mathematical expression that involves symbols (like x) and operations (like addition and multiplication), but does not involve any specific numerical values. In the context of a polynomial ring, a formal expression refers to a polynomial with coefficients in a given field, such as the field of real numbers. The coefficients can be any real number, but they are not assigned specific values until they are evaluated at a specific value of x . For example, the polynomial $p(x) = 2x^2 + 3x - 1$ is a formal expression in the polynomial ring over the field of real numbers. It represents a family of polynomials with coefficients in the real numbers, where the coefficient of x^2 is 2, the coefficient of x is 3, and the constant term is -1. The polynomial can be evaluated at any real value of x , and the resulting value will depend on the choice of x .

Again I am not happy because the programme explains what I ask for by repeating and by using language and notation for objects that do not (yet) exist in our conversation. Everything seems very intuitive and not formally precise.

Rebecca: This is not satisfactory because you use the context of an object that you have not proven exists.

ChatGPT: You're correct, I apologize for the confusion. To provide a clearer answer: In the context of abstract algebra, a formal expression is a string of symbols (called variables) and operations (such as addition and multiplication) that is manipulated according to specific rules. The variables in a formal expression are not assigned any specific value, but are used to represent arbitrary elements of a given set. In the case of a polynomial ring over the field of real numbers, a formal expression is a string of symbols of the form $a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, where a_0, a_1, \dots, a_n are elements of the field of real numbers, and x is a variable. This formal expression represents a polynomial function, which takes a value of x and produces a value in the field of real numbers.

The formal expression does not assign any specific values to x or the coefficients a_0, a_1, \dots, a_n . Instead, it represents the entire family of polynomial functions that can be obtained by assigning different values to x and the coefficients. When we add, subtract, or multiply two formal expressions of this form, we are simply performing the corresponding operations on the coefficients, without evaluating the resulting polynomial function at any specific value of x . So, in summary, a formal expression is a string of symbols and operations that represents an abstract mathematical object, such as a polynomial function. The variables and coefficients in the expression are not assigned any specific values, but are used to represent arbitrary elements of a given set.

I can see now that the programme does not “understand” that I would like to see a construction that does not rely on just formal expressions. It seems as if I can just cook up notation and expressions and invent rules. If I have two variables, where do they live? Do they commute? Also, it seems that “polynomial” and “polynomial function” are used interchangeably. Are they?

Rebecca: Is a polynomial the same as a polynomial function?

ChatGPT: No, a polynomial and a polynomial function are not the same thing, although the terms are often used interchangeably in everyday language. A polynomial is a mathematical expression made up of a finite sum of terms, each of which is a product of a constant coefficient and one or more variables raised to non-negative integer powers. For example, the expression $3x^2 + 2x - 1$ is a polynomial in the variable x . A polynomial function, on the other hand, is a function that can be defined by a polynomial. Specifically, a polynomial function is a function of the form $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, where a_0, a_1, \dots, a_n are constants and x is a variable. The value of the function $f(x)$ at a particular value of x is obtained by substituting that value into the polynomial expression for x . So, a polynomial is an expression, while a polynomial function is a mathematical object that can be defined by a polynomial expression. In many cases, people use the terms “polynomial” and “polynomial function” interchangeably, but strictly speaking they are not the same thing.

Strictly speaking? I will now begin to ask ChatGPT about the difference between a polynomial and the induced function. In a lengthy conversation it becomes clear that the programme only

gives an intuitive description of both and can therefore not explain the difference. I kept thinking: Are you kidding me? Even after several follow-up questions, it does not seem to “get the point”. But see for yourself:

Rebecca: Give an example where a polynomial and a corresponding function are different.

ChatGPT: A polynomial is a mathematical expression consisting of variables and coefficients, and only involves addition, subtraction, and multiplication. A corresponding function is a rule that assigns an output value to each input value. One example where a polynomial and a corresponding function can be different is when the polynomial has a variable that is not present in the function. For instance, consider the polynomial $p(x) = 3x^2 + 2x + 1$. This polynomial is a second-degree polynomial in x and can be evaluated for different values of x . However, if we define a function $f(y) = 3y^2 + 2y + 1$, the input variable is y , not x . The function $f(y)$ assigns an output value to each input value of y , but it is not the same as the polynomial $p(x)$. In other words, $p(x)$ and $f(y)$ are different mathematical expressions, even though they share the same coefficients and exponents. The variable in the polynomial is x , while the variable in the function is y , and they represent different mathematical objects.

Rebecca: x and y is just notation, where is the difference?

ChatGPT: You're correct that the variables x and y are just notations used to represent input values in mathematical expressions, and they don't have any inherent meaning or physical significance. However, when we define a polynomial and a corresponding function with different variables, we are essentially defining two different mathematical objects. In the example I gave earlier, the polynomial $p(x) = 3x^2 + 2x + 1$ and the function $f(y) = 3y^2 + 2y + 1$ are different mathematical expressions because they involve different variables. The polynomial $p(x)$ is defined as a second-degree polynomial in x , which means that its terms involve powers of x up to x^2 . On the other hand, the function $f(y)$ is defined as a second-degree polynomial in y , which means that its terms involve powers of y up to y^2 . Even though x and y are just notations, they represent different input variables in the two expressions. If we plug in a value of x into the polynomial $p(x)$, we get a different output than if we plug in the same value into the function $f(y)$. For example, if we plug in $x=2$ into $p(x)$, we get $p(2) = 3(2)^2 + 2(2) + 1 = 15$. But if we plug in the same value into $f(y)$, we get $f(2) = 3(2)^2 + 2(2) + 1 = 15$, which is the same output. Therefore, while x and y are just notations, they represent different input variables in the polynomial and the corresponding function, and hence the expressions are different mathematical objects.

Rebecca: Give an example where a polynomial and the corresponding function are not the same although the same variable is used.

ChatGPT: A polynomial and a corresponding function can be different even if the same variable is used if the polynomial is not defined for certain input values, while the function is defined for all input values. For example, consider the polynomial $p(x) = 1/x$. This polynomial is not defined for $x=0$, since division by zero is undefined. On the other hand, we can define a corresponding function $f(x) = 1/x$ that is defined for all nonzero input values. The function $f(x)$ assigns an output value to each input value of x , but the polynomial $p(x)$ is undefined for $x=0$. In other words, $p(x)$ and $f(x)$ are different mathematical expressions, even though they share the same variable x . The polynomial $p(x)$ is not defined for $x=0$, while the function $f(x)$ is defined for all

nonzero input values. This illustrates the fact that the same variable can represent different mathematical objects, depending on the context in which it is used.

Rebecca: The polynomial $1/x$ is not an element of the polynomial ring over the real numbers.

ChatGPT: You are correct. The polynomial $1/x$ is not an element of the polynomial ring over the real numbers, because it is not a polynomial in the variable x . The polynomial ring over the real numbers, denoted by $R[x]$, consists of all polynomials with real coefficients in the variable x . A polynomial is defined as a finite sum of terms of the form $a_n x^n$, where a_n is a real coefficient and n is a non-negative integer. The expression $1/x$ cannot be expressed in this form as a polynomial in x , because it involves a negative power of x . Polynomials are only defined for non-negative integer powers of x . However, we can define a function $f(x) = 1/x$ that is defined for all nonzero real numbers x . This function is not a polynomial, but it is a well-defined function that can be evaluated for any nonzero real number. Therefore, while the expression $1/x$ is not an element of the polynomial ring over the real numbers, we can still define a corresponding function that is defined for all nonzero real numbers.

Rebecca: You have still not answered my initial question.

ChatGPT: I apologize for the confusion earlier. You are correct that my previous response did not directly address your initial question. To provide an example where a polynomial and a corresponding function are not the same even when the same variable is used, we can consider the polynomial $p(x) = x^2 - 4x + 4$ and the function $f(x) = \sqrt{x^2 - 4x + 4}$. The polynomial $p(x)$ is a quadratic polynomial in x , which can be evaluated for any real value of x . On the other hand, the function $f(x)$ is defined only for values of x that satisfy the inequality $x^2 - 4x + 4 \geq 0$, which can be written as $(x - 2)^2 \geq 0$. This inequality is true for all real values of x , except for $x = 2$. Therefore, the polynomial $p(x)$ and the function $f(x)$ are not the same, even though they involve the same variable x . The function $f(x)$ is defined only for a subset of the real numbers, while the polynomial $p(x)$ is defined for all real numbers.

Are you also getting annoyed? I certainly do. The apologetic tone in combination with the fact that I seem not to get my point across. In order to get an answer to my question, it seems that I have to ask the question so precisely that the answer is almost there already. This means that if I was asking these questions because I wanted to learn about polynomial rings, I might get quite a few confusing ideas and false impressions about what polynomials are. One last try.

Rebecca: The polynomial and the function are not the same! Do you not understand my question?

ChatGPT: I apologize for the confusion earlier. Here's an example where a polynomial and a corresponding function are not the same: Consider the polynomial $p(x) = x^3 - 3x$ and the function $f(x) = \sin(x)$. Even though both $p(x)$ and $f(x)$ are defined for all real values of x , they are different mathematical objects. The polynomial $p(x)$ is a third-degree polynomial in x , while the function $f(x)$ is a trigonometric function. One way to see that they are different is by evaluating their derivatives. The derivative of $p(x)$ is $p'(x) = 3x^2 - 3$, which is a second-degree polynomial. The derivative of $f(x)$ is $f'(x) = \cos(x)$, which is a different function altogether. Furthermore, the polynomial $p(x)$ takes on both positive and negative values for different values of x , while the function $f(x)$ is always between -1 and 1. Therefore, even though both $p(x)$ and $f(x)$ are defined

for all real values of x , they are different mathematical objects because they have different functional forms and behave differently under differentiation and evaluation.

Ok, I give up. Maybe it works abstractly?

Rebecca: Explain the difference, in concept, between a polynomial and a function.

ChatGPT: A polynomial is a type of mathematical expression that consists of a finite sum of terms, each of which is a constant or a variable raised to a non-negative integer power. For example, the polynomial $3x^2 - 4x + 1$ is a polynomial in the variable x , where the terms are $3x^2$, $-4x$, and 1 . On the other hand, a function is a mathematical object that relates an input value to an output value. A function takes an input value, applies a rule to it, and produces an output value. For example, the function $f(x) = 3x^2 - 4x + 1$ takes an input value x , squares it, multiplies it by 3, subtracts 4 times x , adds 1, and produces an output value. The key difference between a polynomial and a function is that a polynomial is a specific type of mathematical expression that involves a finite sum of terms, while a function is a more general mathematical concept that describes a rule for relating input and output values. Polynomials can be thought of as a specific type of function where the rule for relating input and output values is a polynomial expression. However, not all functions can be expressed as polynomials, and not all polynomials represent valid functions. For example, the polynomial $1/x$ cannot be expressed as a polynomial with non-negative integer powers of x , but it still defines a valid function (except at $x=0$). Therefore, while polynomials and functions are related mathematical concepts, they have different definitions and properties, and should not be used interchangeably.

Hm. Mixed feelings. No abstract concept of a function, the description is intuitive. Again, if I was a student, I would get very misleading ideas. Now I want to change the subject. But first I should make it clear that I want short mathematical language. The next topic will be rings with zero divisors, because this is where the original conversation with my students started.

Rebecca: Can you give answers in short mathematical language?

ChatGPT: Yes, I can give answers in short mathematical language. However, please note that some concepts may require more elaborate explanations than a simple mathematical expression can provide.

Rebecca: Answer like a mathematician. Keep it short!

ChatGPT: Sure, I'll do my best to answer concisely.

Rebecca: Tell me about the ring of integers and its factor rings. Which ones do have zero divisors?

ChatGPT: The ring of integers is denoted by \mathbb{Z} and consists of the set of all integers with addition and multiplication as its operations. The factor rings of \mathbb{Z} are of the form $\mathbb{Z}/n\mathbb{Z}$, where n is a positive integer. The factor rings $\mathbb{Z}/n\mathbb{Z}$ have zero divisors if and only if n is not prime. If n is not prime, then there exist non-zero elements in $\mathbb{Z}/n\mathbb{Z}$ whose product is equal to zero. If n is prime, then every non-zero element in $\mathbb{Z}/n\mathbb{Z}$ is invertible, meaning that it has a multiplicative inverse in the same factor ring, and thus, there are no zero divisors.

Hm. To my knowledge, 0, 1 and -1 are not prime, but the factor rings modulo the ideals generated by them do not have zero divisors. Hence there are at least two false statements in this previous response. Will the programme find the mistake?

Rebecca: Is 1 a prime?

ChatGPT: No, 1 is not a prime number. By definition, a prime number is a positive integer greater than 1 that is divisible by only 1 and itself. However, 1 is divisible by only 1 and itself, so it does not meet the definition of a prime number.

Rebecca: Does $\mathbb{Z}/1\mathbb{Z}$ have zero divisors?

ChatGPT: No, $\mathbb{Z}/1\mathbb{Z}$ does not have any zero divisors. This is because $\mathbb{Z}/1\mathbb{Z}$ is isomorphic to the trivial ring $\{0\}$, which contains only a single element, namely 0. In this ring, 0 is the only element, so there are no non-zero elements whose product is equal to zero. Therefore, $\mathbb{Z}/1\mathbb{Z}$ does not have any zero divisors.

So far so good. Let us see whether it spots the mistake now.

Rebecca: This means that one of your earlier statements is false. Please correct.

ChatGPT: You are correct, I apologize for the error in my earlier statement. I misspoke when I said that "the factor rings of \mathbb{Z} are of the form $\mathbb{Z}/n\mathbb{Z}$, where n is a positive integer." The correct statement is that "the factor rings of \mathbb{Z} are of the form $\mathbb{Z}/n\mathbb{Z}$, where n is a positive integer greater than 1." So, if $n = 1$, then $\mathbb{Z}/n\mathbb{Z}$ is isomorphic to the trivial ring $\{0\}$, which contains only a single element, namely 0. In this ring, 0 is the only element, so there are no non-zero elements whose product is equal to zero. Therefore, $\mathbb{Z}/1\mathbb{Z}$ does not have any zero divisors. Thank you for bringing this to my attention, and please let me know if you have any further questions.

Oh no, we have replaced the first mistake with another one! But of course, the programme has no understanding of what a ring is, or a factor ring, or an ideal. We will see that it does not find its way out of the rabbit hole it created for itself here. Also, the apologetic tone mixes with arrogance now. In my own anthropomorphising attitude, of course. Still I feel that I want to be polite to the programme.

Rebecca: Your previous answer was false. Please correct!

ChatGPT: I apologize for the confusion, and thank you for bringing this to my attention. I made an error in my previous answer, and I will correct it here: The factor rings of the ring of integers, \mathbb{Z} , are of the form $\mathbb{Z}/n\mathbb{Z}$, where n is a positive integer greater than 1. If n is not prime, then $\mathbb{Z}/n\mathbb{Z}$ has zero divisors. This is because there exist non-zero elements in $\mathbb{Z}/n\mathbb{Z}$ whose product is equal to zero. If n is prime, then $\mathbb{Z}/n\mathbb{Z}$ does not have any zero divisors. This is because every non-zero element in $\mathbb{Z}/n\mathbb{Z}$ is invertible, meaning that it has a multiplicative inverse in the same factor ring, and thus, there are no zero divisors. Thank you for bringing this to my attention, and please let me know if you have any further questions.

Ok, so it cannot catch its mistake here. What about the other one, with zero?

Rebecca: Is 0 a prime?

ChatGPT: No, 0 is not a prime number. By definition, a prime number is a positive integer greater than 1 that is divisible by only 1 and itself. However, 0 is not a positive integer, and it is divisible by all integers, not just 1 and itself. Therefore, 0 is not a prime number.

Rebecca: Does $\mathbb{Z}/0\mathbb{Z}$ have zero divisors?

No, the notation $\mathbb{Z}/0\mathbb{Z}$ is undefined and does not correspond to a valid ring. This is because the ideal generated by 0 in the ring of integers, \mathbb{Z} , is the set $\{0\}$, which is not a proper ideal (since it contains only one element), and therefore it cannot be used to form a factor ring. In other words, it is not possible to divide \mathbb{Z} by the ideal generated by 0, and therefore the notation $\mathbb{Z}/0\mathbb{Z}$ does not make sense in the context of ring theory.

Interesting... So what about the zero ideal? And if there is a factor ring for each ideal, then $\mathbb{Z}/0\mathbb{Z}$ should be defined, right? Why does the programme not “know” this?

Rebecca: So the zero ideal is not an ideal?

ChatGPT: The zero ideal $\{0\}$ is indeed an ideal of the ring of integers, \mathbb{Z} . In fact, it is the smallest ideal of \mathbb{Z} , since it contains only the zero element and is closed under addition and subtraction. However, the notation $\mathbb{Z}/0\mathbb{Z}$ is not used in ring theory, because the ideal generated by 0 in \mathbb{Z} is just the zero ideal $\{0\}$, which is not a proper ideal. The factor rings of \mathbb{Z} are of the form $\mathbb{Z}/n\mathbb{Z}$, where n is a positive integer greater than 1. In summary, the zero ideal is a valid ideal of \mathbb{Z} , but the notation $\mathbb{Z}/0\mathbb{Z}$ is not used in ring theory because it does not correspond to a valid factor ring.

Rebecca: This is false. $\mathbb{Z}/\{0\}$ is a ring.

ChatGPT: You are correct, and I apologize for my mistake in my previous response. The ideal $\{0\}$ is a valid ideal of the ring of integers \mathbb{Z} , and the quotient ring $\mathbb{Z}/\{0\}$ is isomorphic to the ring \mathbb{Z} itself. In other words, $\mathbb{Z}/\{0\}$ is just \mathbb{Z} itself, and it does not have any zero divisors since \mathbb{Z} is an integral domain. Therefore, my previous statement that $\mathbb{Z}/0\mathbb{Z}$ is undefined and does not correspond to a valid ring is incorrect. Thank you for bringing this to my attention, and please let me know if you have any further questions.

Interesting. I keep correcting it, but it tells me that it is ok to ask further questions. Oh well.

Of course I will, because I find it fascinating to read the answers and to spot the mistakes and try to understand where they come from.

Rebecca: One of your earlier "if and only if" statements about factor rings of \mathbb{Z} and their zero divisors is false. Please correct.

ChatGPT: I apologize for any confusion I may have caused. To clarify, here is the correct statement: The factor rings of the ring of integers, \mathbb{Z} , are of the form $\mathbb{Z}/n\mathbb{Z}$, where n is a positive integer greater than 1. If n is not prime, then $\mathbb{Z}/n\mathbb{Z}$ has zero divisors. This is because there exist non-zero elements in $\mathbb{Z}/n\mathbb{Z}$ whose product is equal to zero. If n is prime, then $\mathbb{Z}/n\mathbb{Z}$ does not have any zero divisors. This is because every non-zero element in $\mathbb{Z}/n\mathbb{Z}$ is invertible, meaning that it has a multiplicative inverse in the same factor ring, and thus, there are no zero divisors. Thank you for bringing this to my attention, and please let me know if you have any further questions.

Hm. The same mistake, over and over again. And new ones, because it forgets some interesting ideals. Maybe we try it again, pointing it directly to it?

Rebecca: "If n is not prime, then $\mathbb{Z}/n\mathbb{Z}$ has zero divisors. " This is false. Please correct.

ChatGPT: I apologize for the confusion. You are correct, my previous statement is not entirely accurate. While it is true that if n is not prime, then $\mathbb{Z}/n\mathbb{Z}$ may have zero divisors, it is not necessarily the case. In fact, there exist composite integers n such that $\mathbb{Z}/n\mathbb{Z}$ is an integral domain, meaning it does not have any zero divisors. For example, if $n = 6$, then $\mathbb{Z}/6\mathbb{Z} \cong \mathbb{Z}_2 \oplus \mathbb{Z}_3$, which is an integral domain. On the other hand, if $n = 4$, then $\mathbb{Z}/4\mathbb{Z} \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$, which does have zero divisors. Therefore, the correct statement is: If n is a composite integer, then $\mathbb{Z}/n\mathbb{Z}$ may or may not have zero divisors, depending on the prime factorization of n . I apologize for any confusion my earlier statement may have caused, and thank you for bringing it to my attention.

Thanks for playing, but confusion is not what has been caused. I am amazed by how many new wrong statements have been produced, and I have to give a serious warning to my students about using this programme for studying. All the positive, useful things I have found so far, around generating ideas, have been far away from mathematical content. I wonder whether I can find something here that works, as inspiration for me and my students?

Rebecca: Give three exercise related to integral domains.

ChatGPT:

1. Show that the ring $\mathbb{Z}[i]$ of Gaussian integers is an integral domain.
2. Prove that any finite integral domain is a field.
3. Let R be an integral domain and let a, b be elements of R . If $a \mid b$ and $b \mid a$, then show that a and b are associates (i.e., differ only by a unit of R).

Rebecca: Explain what the students learn with these exercises.

ChatGPT: These exercises help students develop a deeper understanding of the properties of integral domains, including the relationship between divisibility and the existence of zero divisors,

and the connection between integral domains and fields. Through these exercises, students will also gain experience with proof techniques, including direct proof, proof by contradiction, and proof by contrapositive. Additionally, these exercises will help students develop their problem-solving skills and ability to think critically about abstract algebraic structures.

This is a bit generic I think. The very first one would not be an exercise with me because the Gaussian integers are a subring of the field of complex numbers, and hence they are clearly an integral domain. I might use it for the students to practice proving that a set is a ring, but even there they have enough tools for this to be an easy exercise.

Rebecca: The first exercise follows directly from the fact that fields are integral domains. What exactly do the students learn here?

ChatGPT: You are correct that the first exercise can be shown by noting that the ring of Gaussian integers is a field. However, this exercise is still useful for students as it provides them with practice in verifying the properties of integral domains, particularly in the context of Gaussian integers. It also helps to illustrate the importance of understanding the properties of fields and integral domains in abstract algebra, since many other algebraic structures can be understood in terms of these fundamental concepts.

Oops!? A clearly wrong statement, and then a lot of general stuff. I guess that I can get inspiration for exercises, but I have to check everything carefully. One more try?

Rebecca: Give three exercises about integral domains.

ChatGPT: Here are three exercises about integral domains:

1. Let R be an integral domain and let $a, b, c \in R$. Prove that if $a \mid bc$ and a is relatively prime to b , then $a \mid c$.
2. Let R be an integral domain and let $a, b, c \in R$. Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.
3. Let R be an integral domain and let $a, b, c \in R$. Prove that if $ab = ac$ and $a \neq 0$, then $b = c$.

Note: In each of these exercises, you should use the definition of integral domain and any relevant properties of integral domains that you know. You may also need to use some basic properties of divisibility and equality.

I would advise my students to be sceptical of the first exercise, because they know integral domains where there is no GCD in general, and hence there is no concept like “relatively prime” in general there, as well.

Rebecca: Is "relatively prime" defined in every integral domain?

ChatGPT: No, "relatively prime" is not defined in every integral domain. The concept of "relatively prime" makes sense for integral domains that have a well-defined notion of greatest common divisor (GCD). An integral domain R is said to have a GCD if for any two non-zero elements $a, b \in R$, there exist elements $d, x, y \in R$ such that d is a GCD of a and b (i.e., d divides both a and b , and any common divisor of a and b divides d), and $ax + by = d$. In integral domains that have a GCD, two elements a and b are said to be relatively prime if their GCD is 1. In this case, there exist elements x and y such that $ax + by = 1$, which implies that a and b are not

zero divisors of each other. However, there are integral domains that do not have a GCD, such as the polynomial ring $F[x]$ over a field F . In such domains, the concept of "relatively prime" is not well-defined.

Oops! Polynomial rings are Euclidean, of course we have GCDs there!

Rebecca: The last statement is wrong, please correct.

ChatGPT: I apologize for my previous response. The statement "there are integral domains that do not have a GCD" is incorrect, as every Euclidean domain (which includes many important examples of integral domains such as the ring of integers and polynomial rings over fields) has a GCD. Therefore, in any Euclidean domain, including the examples mentioned above, the concept of "relatively prime" is well-defined as two elements a and b are said to be relatively prime if their greatest common divisor is 1. Thank you for bringing this to my attention, and please let me know if you have any further questions.

Again, things are getting worse rather than better. I am beginning to think that I should use conversations like this and give pointers to the students, so they find and correct the mistakes. This might actually be really useful, and I am sure that if I ask the programme to give me 10, 20 or 50 such little exercises or examples, then it will produce them.

Finally, we go to my research area. I want to see what ChatGPT will tell me about finite simple groups, in particular their classification. Does it know the relevant definitions and the statement of the classification theorem? Will it be able to give references? I wonder whether it can help, for example, with writing brief introductory texts for a talk or the introduction to an overview article.

Rebecca: What is a finite simple group?

ChatGPT: A finite simple group is a non-trivial group that has no non-trivial normal subgroups, and is finite in size. In other words, a finite simple group is a group that cannot be decomposed into smaller groups, other than the trivial group and itself. Examples of finite simple groups include the cyclic groups of prime order, the alternating groups of degree at least 5, and the groups of Lie type over finite fields. The classification of finite simple groups is a major achievement in group theory, and it states that every finite simple group can be classified into one of several families, including the cyclic groups of prime order, the alternating groups, the sporadic groups, and the groups of Lie type.

Again, lots of things I did not ask for, but let us move on.

Rebecca: Explain briefly what the classification of finite simple groups is.

ChatGPT: The classification of finite simple groups is a monumental achievement in the field of mathematics, which classifies all finite simple groups into several broad categories. A finite simple group is a finite group that does not contain any nontrivial normal subgroups, and these groups are the building blocks of finite group theory. The classification theorem states that every finite simple group belongs to one of 18 broad categories, known as the "sporadic" groups, or one of the families of "classical" or "exceptional" groups. The classical groups include the finite versions of the classical Lie groups from algebraic geometry, while the exceptional groups are more exotic and have no classical counterparts. The proof of the classification theorem is extremely complex and involved the work of many mathematicians over several decades. The classification theorem is considered one of the most important achievements in mathematics of the 20th century, as it provides a complete understanding of the structure of all finite simple groups. It has had

numerous applications in areas such as algebraic geometry, number theory, combinatorics, and theoretical physics.

Wait. 18 categories? I have never seen it explained in this way and I wonder what ChatGPT means.

Rebecca: Describe the 18 broad categories.

ChatGPT: The 18 broad categories of finite simple groups in the classification theorem are:

1. The cyclic groups of prime order
2. The alternating groups of degree at least 5
3. The groups of Lie type in characteristic 2
4. The Suzuki groups
5. The Ree groups
6. The groups of Lie type in odd characteristic
7. The Tits group
8. The sporadic group known as the Mathieu group
9. The sporadic group known as the Higman-Sims group
10. The sporadic group known as the McLaughlin group
11. The sporadic group known as the Suzuki sporadic group
12. The sporadic group known as the Ree sporadic group
13. The sporadic group known as the Conway group
14. The sporadic group known as the Co₁ sporadic group
15. The sporadic group known as the Co₂ sporadic group
16. The sporadic group known as the Co₃ sporadic group
17. The sporadic group known as the H₁ sporadic group
18. The sporadic group known as the H₂ sporadic group

These groups differ in their underlying mathematical structures and have different properties, which are studied extensively in group theory and other areas of mathematics. The classification theorem provides a complete understanding of the structure of all finite simple groups, and its proof is considered one of the greatest achievements in mathematics.

This is at least unusual and needs some explanation. What are H₁ and H₂? Why are the Conway groups split up in this way?

Rebecca: Why do you differentiate "the Conway group" in item 13 from items 14-16?

ChatGPT: I apologize for the confusion in my previous response. The sporadic group known as the Conway group should actually be included in the Co₁, Co₂, and Co₃ sporadic groups, which are sporadic groups 14, 15, and 16 in the list of 18 sporadic groups. The Conway group is a subgroup of the Co₁ sporadic group and is sometimes referred to as the "Co₁ subgroup of the Conway group." The reason for this differentiation is that the Co₁, Co₂, and Co₃ sporadic groups are closely related and share certain properties, such as being constructed from certain exceptional mathematical objects known as "sporadic simple groups" and "Steiner systems." The Conway group is also constructed using similar methods, but it is not a member of the Co₁, Co₂, or Co₃ sporadic groups. I apologize for any confusion my previous response may have caused and hope this clarifies the distinction between the Conway group and the Co₁, Co₂, and Co₃ sporadic groups.

I am even more confused now. But there are more questions about the 18 categories that I want to ask.

Rebecca: Why does item 8 refer to only one group that you call "Mathieu group"?

ChatGPT: You are correct, my apologies for the error. The sporadic group known as the Mathieu group should actually be referred to in the plural form as the "Mathieu groups." There are actually several Mathieu groups, denoted by M_{11} , M_{12} , M_{22} , M_{23} , M_{24} , which are all sporadic groups. The Mathieu groups are named after the French mathematician Émile Léonard Mathieu, who first discovered these groups in the late 19th century. They are interesting and important examples of finite simple groups, and they have many interesting mathematical properties. Again, I apologize for any confusion caused by my previous response, and I hope this clarifies the matter.

Rebecca: Please explain exactly how the 26 sporadic simple groups fit into the items that you listed.

ChatGPT: I apologize for any confusion caused by my previous responses. To clarify, the sporadic simple groups are a subset of the sporadic groups and are a crucial part of the classification of finite simple groups. The 26 sporadic simple groups are:

1. The Mathieu groups M_{11} , M_{12} , M_{22} , M_{23} , M_{24}
2. The Higman-Sims group
3. The McLaughlin group
4. The Conway group
5. The Co_1 , Co_2 , and Co_3 sporadic groups
6. The Janko groups J_1 , J_2 , J_3 , J_4 , J_5 , J_6 , J_7 , J_8 , and J_9
7. The Fischer groups F_{22} , F_{23} , and F_{24}

The Mathieu groups are the first five sporadic simple groups and are named after the French mathematician Émile Léonard Mathieu. They are related to the geometry of certain finite geometries and have important applications in combinatorics. The Higman-Sims group, McLaughlin group, Conway group, and Co_1 , Co_2 , and Co_3 sporadic groups are the next six sporadic simple groups and are closely related. They are constructed from certain mathematical objects known as "Steiner systems" and "sporadic simple groups." The Conway group is actually a subgroup of the Co_1 sporadic group, as previously mentioned. The Janko groups are named after the mathematician Zvonimir Janko and are a family of 9 sporadic simple groups. They have important applications in the theory of finite fields and algebraic geometry. Finally, the Fischer groups are a family of 3 sporadic simple groups and are named after the mathematician Bernd Fischer. They were discovered using a computer search and have interesting mathematical properties. All 26 sporadic simple groups are important and interesting objects of study in group theory and have many applications in other areas of mathematics and science.

I do not understand the notation and how the programme organises the groups. There seem too many Janko groups! Maybe I can ask where the notation comes from, or ask for a reference.

Rebecca: Give a reference for your notation for the groups.

ChatGPT: The notation for the finite simple groups that I used in my previous responses is based on the standard notation used in the mathematical literature. The notation was first

introduced in the 1950s and 1960s as part of the effort to classify all finite simple groups. It has since become the standard way of referring to these groups in the mathematical literature. The notation typically consists of a letter or a combination of letters, followed by one or more subscripts. The letter(s) denote the family of groups to which the group belongs, while the subscripts provide additional information about the group's structure and properties. A standard reference for the notation and properties of the finite simple groups is the book "The Finite Simple Groups" by Robert Wilson, published by Springer-Verlag in 2009. Another useful reference is the book "Finite Simple Groups: An Introduction to their Classification" by Daniel Gorenstein, published by Plenum Press in 1982.

Of course, the programme does not know that I know some things about finite groups and that I am familiar with at least some of the standard notation. Therefore, the next piece of our conversation made me laugh a lot.

Rebecca: I have not seen this notation. Be more specific.

ChatGPT: I apologize for the confusion. The notation I used for the finite simple groups is based on the standard notation used in the mathematical literature, and it may not be immediately familiar to everyone. The notation typically consists of a letter or a combination of letters, followed by one or more subscripts. The letter(s) denote the family of groups to which the group belongs, while the subscripts provide additional information about the group's structure and properties. Here is a more specific description of the notation for the finite simple groups that I used in my previous responses:

1. The cyclic groups of prime order are denoted by the symbol C_p , where p is a prime number.
2. The alternating groups are denoted by the symbol A_n , where n is the number of elements in the group.
3. The family of Lie-type groups are denoted by the symbols $L_n(q)$, where n is a positive integer and q is a prime power.
4. The family of twisted Lie-type groups are denoted by the symbols ${}^2B_2(q)$, ${}^2G_2(q)$, ${}^3D_4(q)$, ${}^2F_4(q)$, ${}^2E_6(q)$, ${}^2E_7(q)$, and ${}^2E_8(q)$, where q is a prime power.
5. The family of sporadic groups are denoted by the symbols M_n , where n is a positive integer, and the symbols J_n , F_{22} , F_{23} , F_{24} , and the Conway groups Co_1 , Co_2 , and Co_3 .

I hope this more specific description of the notation is helpful.

Rebecca: This is not helpful. What is J_5 ?

ChatGPT: I apologize for my previous responses not being clear enough. The group J_5 is one of the sporadic simple groups, which is also known as the Held group. It has order $2^9 \cdot 3^4 \cdot 5 \cdot 7 \cdot 11 \cdot 19 \cdot 31 \cdot 61 \cdot 151$, and was discovered in 1965 by the mathematician Klaus Friedrichs. The Held group is the smallest of the five sporadic groups that are sometimes called the "pariahs" because they do not fit into any of the other sporadic group families. The Held group has important applications in coding theory and the design of error-correcting codes.

The programme is making stuff up now. But it does not "know" that I know, and I will just keep asking a bit more to see what else it will come up with.

Rebecca: Why is it called Held group and denoted with a J if it was discovered by Klaus Friedrichs?

ChatGPT: I apologize for the confusion. The Held group is actually denoted by the symbol J_2 , not J_5 . I must have made a mistake in my previous response. The Held group J_2 is named after its discoverer, the mathematician Adolf H. Held. It has order $2^6 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 19$, and it is one of the five sporadic groups that are sometimes called the "pariahs" because they do not fit into any of the other sporadic group families. The group J_2 has important applications in coding theory and the design of error-correcting codes. I apologize for any confusion my earlier responses may have caused.

Rebecca: This is false, J_2 does not denote the Held group. Please correct.

ChatGPT: I apologize for my mistake in my previous response. The Held group is actually denoted by the symbol J_1 , not J_2 . The group J_1 is named after its discoverer, the mathematician Adolf H. Held. It has order $2^{15} \cdot 3^6 \cdot 5^3 \cdot 7^2 \cdot 11 \cdot 19 \cdot 31 \cdot 47 \cdot 67$, and it is one of the five sporadic groups that are sometimes called the "pariahs" because they do not fit into any of the other sporadic group families. The group J_1 has important applications in coding theory and the design of error-correcting codes. I apologize for any confusion my earlier responses may have caused.

No need to apologise, I had a lot of fun reading this nonsense. No idea who Klaus Friedrichs is, and Dieter Held will be amused about his new first name Adolf and the fact that he discovered the first Janko group. No more questions from me right now!