Lösung zur Klausur „CII" vom 27. Jul. 2026

1| $f(u,v) = \frac{u^3}{v} + \sin(u \cdot \ln u)$

Satz von Schwarz

$$f_v = -\frac{u^3}{v^2}, \quad f_{vuu} = -\frac{6u}{v^2} = f_{uuv}$$

2| $\int_\Gamma P\,dx + Q\,dy$ mit $P = ye^x + 2x$, $Q = e^x$

1) $P_y = e^x$, $Q_x = e^x \Rightarrow$ wegunabhängig

2) $F = \int P\,dx = ye^x + x^2 + C(y)$, $F_y = e^x + C'(y) \overset{!}{=} e^x$

$\Rightarrow C(y) = K \in \mathbb{R}$

$\Rightarrow$ $\underline{F = ye^x + x^2}$ ist Stammfunktion

3) $\int_\Gamma P\,dx + Q\,dy = F\Big|_{(0,0)}^{(1,1)} = \underline{e+1}$

3| $L = (x+y)y + \lambda(x^2+y^2-1)$

$$\left.\begin{array}{lll} \text{I} & L_x = y + 2\lambda x = 0 \\ \text{II} & L_y = x + 2y + 2\lambda y = 0 \end{array}\right\} \Rightarrow (x\,\text{I} - y\,\text{II}):\; x^2 + 2xy - y^2 = 0$$

$\text{III} \quad L_\lambda = x^2 + y^2 - 1$

$\Rightarrow$ Ansatz: $y = \alpha x$

$\Rightarrow x^2 + 2x\alpha x - \alpha^2 x^2 = 0 \quad | : x^2$

$\Rightarrow 1 + 2\alpha - \alpha^2 = 0$

$\Rightarrow \alpha^2 - 2\alpha - 1 = 0 \Rightarrow \alpha_{1/2} = 1 \pm \sqrt{1+1}$

$= 1 \pm \sqrt{2}$

Aus III folgt nun:

$$x^2 + \alpha^2 x^2 = 1 \Leftrightarrow x^2 = \frac{1}{1+\alpha^2} \Leftrightarrow x_{1/2} = \frac{\pm 1}{\sqrt{1+\alpha^2}}$$

$\Rightarrow (x,y) \in \{(-0.382, -0.923), (0.382, 0.923), (-0.923, 0.382), (0.923, 0.382)\}$

4) $\left\langle \begin{pmatrix} 2 \\ \alpha \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ -\alpha \\ 2 \end{pmatrix} \right\rangle = 2+6-4\alpha \overset{!}{=} 0$

$\Rightarrow \underline{\underline{\alpha = 2}}$

5) $\frac{d}{dt} -2(\cos t)^2 - \frac{1}{2}t^2 \Big|_{t=1} = +4\cos t \sin t - t \Big|_{t=1}$

$= 4\cos(1)\sin(1) - 1$

$= 2\sin(2) - 1 = 0.818\ldots$

6)
$$\left.\begin{matrix} 1 & 2 & -1 & 3 \\ 2 & 4 & -2 & 6 \\ 0 & 1 & 1 & 1 \\ 1 & 3 & 0 & 4 \end{matrix}\right| \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} \rightarrow \left.\begin{matrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{matrix}\right| \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} \rightarrow \left(\left.\begin{matrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}\right| \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}\right)$$

$\Rightarrow$ Rang $= 2$

$\Rightarrow x_4 = s,\ x_3 = t$

$\Rightarrow x_2 = -s-t$

$x_1 = -3s + t - 2(-s-t) = -s + 3t$

$\Rightarrow x = t\begin{pmatrix} 3 \\ -1 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$, $\underline{\underline{\text{Ker}(A) = \text{Span}\left(\begin{pmatrix} 3 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix}\right)}}$

7) $\det\begin{pmatrix} 2-\lambda & a \\ 1 & 7-\lambda \end{pmatrix} = \lambda^2 - 9\lambda + 14 - a$. $\lambda_1 + \lambda_2 = 9$ und $\lambda_2 = 2\lambda_1$

$\Rightarrow \lambda_1 = 3,\ \lambda_2 = 6,\ \Rightarrow (14-a) = \lambda_1 \cdot \lambda_2 = 18 \Rightarrow \underline{\underline{a = -4}}$