6. Feb. 2015

Lösung für die Klausur vom 6. Feb.

Aufgabe 1 a) $\dfrac{50}{(3+i)^2} = \dfrac{50}{8+6i} = \dfrac{1}{100}\,50(8-6i) = \underline{\underline{4-3i}}$

b) $\varphi = \arctan\left(\frac{1}{3}\right) = 0.321751$, $r = \sqrt{10} = 3.16228$

$\underline{\underline{w_0 = 1.4678\, e^{0.10725 i}}}$, $\underline{\underline{w_1 = w_0 \cdot e^{\frac{2\pi i}{3}}}}$, $\underline{\underline{w_2 = w_0\, e^{\frac{4\pi i}{3}}}}$

Zusatz: $\ln(3+i) = \ln\left(\sqrt{10}\, e^{i \arctan\left(\frac{1}{3}\right)}\right) = \frac{1}{2}\ln 10 + i \arctan\left(\frac{1}{3}\right)$

$= \underline{\underline{1.15129 + 0.321751\, i}}$

Aufgabe 2 a) $\lim\limits_{x\to\pi} \dfrac{\tan(x)}{\sin(x)} = \lim\limits_{x\to\pi} \dfrac{1}{\cos(x)} = \underline{\underline{-1}}$

b) $\lim\limits_{x\to\infty} \dfrac{\ln(x^2) - \ln(\sqrt{x})}{\ln(x)} = \lim\limits_{x\to\infty} \dfrac{2\cancel{\ln x} - \frac{1}{2}\cancel{\ln x}}{\cancel{\ln x}} = \underline{\underline{\frac{3}{2}}}$

Aufgabe 3 $f'(x) = e^{\sqrt{x}} \cdot \frac{1}{2}\frac{1}{\sqrt{x}} - 2x$, $\varphi = x - \dfrac{f(x)}{f'(x)}$

$x_0 = 80$, $x_1 = \underline{\underline{75.2915}}$, $x_2 = \underline{\underline{74.2354}}$, $x_3 = \underline{\underline{74.1858}}$

Aufgabe 4 $\int \underset{u}{x}\, \underset{v'}{e^{2-2x}}\, dx = x\, e^{2-2x} \cdot \frac{1}{-2} + \frac{1}{2}\int e^{2-2x}\, dx = -\frac{1}{4} e^{2-2x}(1+2x)$

$\Rightarrow \int_0^\infty x\, e^{2-2x}\, dx = -\frac{1}{4} e^{2-2x}(1+2x)\Big|_0^\infty = \dfrac{e^2}{4} = \underline{\underline{1.84726}}$

Aufgabe 5 $y' = \dfrac{y^3}{1-y}\, t \Rightarrow \int \dfrac{1-y}{y^3}\, dy = \int t\, dt$

$\Rightarrow -\frac{1}{2} y^{-2} + \frac{1}{y} = \frac{1}{2} t^2 + C$, $y(0) = -1 \Rightarrow C = -\frac{3}{2}$

$\underset{\text{quad. Gleichung}}{\Rightarrow}$ $y(t) = \dfrac{1+\sqrt{4-t^2}}{t^2-3}$, $y(1) = -\dfrac{1+\sqrt{3}}{2} = \underline{\underline{-1.36603}}$