Facility Location Optimizer

A tool for solving location problems.

Classification: $\mathbf{1 \mid P \mid v > 0 \mid \mu_i \mid median}$

---

1. Problem formulation

The goal is to find one new facility $x \in \mathbb{R}^2$ in the plane such that the weighted sum of the distances between the new facility x and given facilities located at the points $a^1, \ldots, a^m \in \mathbb{R}^2$ are minimized. Such problems are called

"Single facility median location problems in the plane"

in the literature of location theory and can be interpreted as a model for minimizing transportation costs.

Let B_{μ_i}, $i = 1, \ldots, m$, be polytopes (closed, bounded and polyhedral sets) in $\mathbb{R}^2$ with $0 \in \operatorname{int} B_{\mu_i}$, $i = 1, \ldots, m$. Using a point-specific polyhedral gauge function (also known as a special case of the Minkowski functional), which is defined by

$$\mu_i(x - a^i) := \inf\{\lambda > 0 \,|\, x - a^i \in \lambda \cdot B_{\mu_i}\}$$

for all $x := (x_1, x_2) \in \mathbb{R}^2$ and all $a^i := (a_1^i, a_2^i)$, $i = 1, \ldots, m$, the location problem is given by

$$\sum_{i=1}^{m} v_i \cdot \mu_i(x - a^i) \to \min_{x \in \mathbb{R}^2},$$

where $v_1, \ldots, v_m \in \mathbb{R}$ are positive weights (e.g. demands of the given facilities).

Summarizing, in our problem

$$1 \mid \mathrm{P} \mid v > 0 \mid \mu_i \mid \text{median}$$

we search for one new facility (position 1: 1) in the plane (position 2: P), the given facilities have positive weights (position 3: $v > 0$, i.e., $v_i > 0$ for all $i = 1, \ldots, m$) and we consider a median problem (position 5: median), where we measure the distances between points using point-specific gauges μ_i, $i = 1, \ldots, m$ (position 4: μ_i).

2. Algorithm information and implementation

The corresponding algorithm included in the current version of the Software FLO generates exact solutions of the above location problem. The program uses the algorithm (Construction Grid Algorithm) proposed in the paper by Durier and Michelot (1985). More information about the procedure can be found for instance in the dissertations of Nickel (1995), Bischoff (2008) or Wagner (2014).

The algorithm was implemented in FLO by Christian Günther. Software FLO has been able to solve the underlying location problem since program version 1.0.0, which was released on 22/04/2015.

3. Selected References

Further model and algorithm-specific information can be found in the following literature:

(A) R. Durier and C. Michelot. *Geometrical properties of the Fermat-Weber problem.* European Journal of Operational Research, 20:332-343, 1985.

(B) S. Nickel. *Discretization of Planar Location Problems.* Verlag Shaker, Aachen, 1995.

(C) M. Bischoff. *Location of Connection Facilities.* Verlag Shaker, Aachen, 2008.

(*D*) A. Wagner. *A new Duality Based Approach for the Problem of Locating a Semi-Obnoxious Facility.* Dissertation, Martin-Luther-Universität Halle-Wittenberg, 2014.