Facility Location Optimizer

A tool for solving location problems.

Classification: $\mathbf{1} \mid \mathbf{P} \mid v > 0,\ X = \mathbb{R}^2 \setminus \operatorname{int} P \mid d_2^2 \mid \textbf{median}$

1. Problem formulation

Let $P \subseteq \mathbb{R}^2$ be a polytope (i.e., a bounded polyhedral set) and assume that we have a representation

$$P = \operatorname{conv}\{e^1, \ldots, e^l\}$$

for a finite number of extreme points $e^1, \ldots, e^l \in \mathbb{R}^2$, $l \in \mathbb{N}$.

Moreover, let the feasible set X be represented by the whole plane but we exclude a special forbidden region defined by the interior of the polytope P, i.e., we have $X = \mathbb{R}^2 \setminus \operatorname{int} P$.

The goal is to find one new facility $x \in X = \mathbb{R}^2 \setminus \operatorname{int} P$ such that the weighted sum of the distances between the new facility x and the given facilities located at the points $a^1, \ldots, a^m \in \mathbb{R}^2$ are minimized. Such problems are called

"Single facility median location problems
in the plane involving a forbidden region"

in the literature of location theory and can be interpreted as a model for minimizing transportation costs.

Using the well-known squared Euclidean metric, which is defined by

$$d_2^2(x, a^i) := (x_1 - a_1^i)^2 + (x_2 - a_2^i)^2$$

for all $x := (x_1, x_2) \in \mathbb{R}^2$ and all $a^i := (a_1^i, a_2^i)$, $i = 1, \ldots, m$, the location problem is given by

$$\sum_{i=1}^{m} v_i \cdot d_2^2(x, a^i) = \sum_{i=1}^{m} v_i \cdot ((x_1 - a_1^i)^2 + (x_2 - a_2^i)^2) \to \min_{x \in X = \mathbb{R}^2 \setminus \operatorname{int} P}$$

with positive weights $v_1, \ldots, v_m \in \mathbb{R}$ (e.g. demands of the given facilities).

Note: Median location problems with positive weights involving a forbidden region are non-convex problems, since $X = \mathbb{R}^2 \setminus \operatorname{int} P$ is a non-convex set in $\mathbb{R}^2$.

Summarizing, in our problem

$$1 \mid \mathrm{P} \mid v > 0,\ X = \mathbb{R}^2 \setminus \operatorname{int} P \mid d_2^2 \mid \text{median}$$

we search for one new facility (position 1: 1) in the plane (position 2: P), the given facilities have positive weights, the search for the new facility is restricted to the feasible set $X = \mathbb{R}^2 \setminus \operatorname{int} P$ (position 3: $v > 0$, i.e., $v_i > 0$ for all $i = 1, \ldots, m$; $X = \mathbb{R}^2 \setminus \operatorname{int} P$) and we consider a median problem (position 5: median), where we measure the distances between points using the squared Euclidean metric (position 4: d_2^2).

2. Algorithm information and implementation

The corresponding algorithm included in the current version of the Software FLO generates the whole set of solutions of the above location problem. The program uses a modified and extended version of the algorithm (Projection Algorithm) formulated in the book by Hamacher (1995).

Note: We assume that the solution set of the free problem "$1 \mid \mathrm{P} \mid v > 0 \mid d_2^2 \mid$ median" is completely contained in the interior of the polytope P such that no solution of the free problem is feasible for the restricted problem "$1 \mid \mathrm{P} \mid v > 0,\ X = \mathbb{R}^2 \setminus \operatorname{int} P \mid d_2^2 \mid$ median".

The algorithm was implemented in FLO by Rico Khan during his Bachelor-Thesis (see Reference A). Software FLO has been able to solve the underlying location problem since version 1.2.0 which was released on 08/01/2016.

3. Selected References

Further model and algorithm-specific information can be found in the following literature:

(A) R. Khan. *Algorithmen für restringierte Standortprobleme einschließlich Implementierung*. Bachelor-Thesis, Martin Luther University Halle-Wittenberg, 2015.

(B) H. W. Hamacher. *Mathematische Lösungsverfahren für planare Standortprobleme*. Vieweg Verlag, 1995.