Facility Location Optimizer

A tool for solving location problems.

Classification: $\mathbf{1} \mid \mathbf{P} \mid (+) \mid \mu \mid$ **Eff-vector**

1. Problem formulation

The goal is to find one new facility $x \in \mathbb{R}^2$ in the plane such that the distances between the new facility x and given facilities located at the points $a^1, \ldots, a^m \in \mathbb{R}^2$ are minimized simultaneously, where simultaneous minimization is understood in the sense of multi-objective optimization. Such problems are called

"Single facility multi-objective location problems in the plane"

in the literature of location theory and can be interpreted as a model for minimizing transportation costs.

Let B be a polytope (closed, bounded and polyhedral set) in $\mathbb{R}^2$ with $0 \in \operatorname{int} B$ and $B = -B$ (symmetry with respect to the origin). Moreover, assume that the polytope B can be represented by the convex hull of four points $r^1, r^2, r^3, r^4 \in \mathbb{R}^2$, i.e., it holds $B = \operatorname{conv}\{r^1, r^2, r^3, r^4\}$.

Now using the unit ball $B_\mu := B$ we can define a block norm μ. Note that the unit ball of the maximum norm (a special block norm) is determined by four extreme points

$$r^1 = (-1, 1), r^2 = (1, 1), r^3 = (1, -1), r^4 = (-1, -1) \in \mathbb{R}^2.$$

The distances induced by the block norm μ are defined by

$$\mu(x - a^i) := \inf\{\lambda > 0 \,|\, x - a^i \in \lambda \cdot B_\mu\}$$

for all $x := (x_1, x_2) \in \mathbb{R}^2$ and all $a^i := (a_1^i, a_2^i)$, $i = 1, \dots, m$. Hence, the location problem is given by

$$\begin{pmatrix} \mu(x - a^1) \\ \dots \\ \mu(x - a^m) \end{pmatrix} \to \underset{x \in X = \mathbb{R}^2}{\text{v-min}} .$$

In this problem, we search for Edgeworth-Pareto efficient solutions.

Summarizing, in our problem

$$1 \mid \text{P} \mid (+) \mid \mu \mid \text{Eff-vector}$$

we search for one new facility (position 1: 1) in the plane (position 2: P), the given facilities are attraction points (position 3: (+)) and we consider a vector problem with regard to the solution concept of Edgeworth-Pareto efficiency (position 5: Eff-vector), where we measure the distances between points using a metric induced by a special class of block norms (position 4: μ).

2. Algorithm information and implementation

The current version of the Software FLO solves the above location problem and computes the whole set of Edgeworth-Pareto efficient solutions. The program solves the location problem using an algorithm (Transformation Algorithm) that is based on the algorithm for solving the problem "1 | P | (+) | μ | Eff-vector" and an appropriate linear transformation (see Günther, Popovici and Tammer, 2016).

The algorithm was implemented in FLO by Christian Günther. Software FLO has been able to solve the underlying location problem since version 1.2.0 which was released on 08/01/2016.

3. Selected References

Further model and algorithm-specific information can be found in the following literature:

(*A*) C. Günther, N. Popovici and Chr. Tammer. *Reducing the complexity of planar multiobjective location problems*, 2016 (in preparation).

(*B*) S. Alzorba, C. Günther, N. Popovici and Chr. Tammer. *A new algorithm for solving planar multiobjective location problems involving the Manhattan norm*. Preprint Optimization-Online, http://www.optimization-online.org/DB_HTML/2016/01/5305.html, 2015 (submitted).