Facility Location Optimizer

A tool for solving location problems.

Classification: 1 | P | $v > 0, w < 0$ | d_∞ | median

1. Problem formulation

The goal is to find one new facility $x \in \mathbb{R}^2$ in the plane such that the weighted sum of the distances between the new facility x and given attration facilities located at the points $a^1, \ldots, a^{m_1} \in \mathbb{R}^2$ as well as given repulsion facilities located at the points $b^1, \ldots, b^{m_2} \in \mathbb{R}^2$ are minimized. Such problems are called

"Single facility median location problems with attraction and repulsion points in the plane"

in the literature of location theory. Using the well-known maximum metric (also called Chebyshev metric or l_∞ metric), which is defined by

$$d_\infty(x, y) := \max\{|x_1 - y_1|, |x_2 - y_2|\}$$

for all $x := (x_1, x_2), y := (y_1, y_2) \in \mathbb{R}^2$, the location problem is given by

$$\left[\sum_{i=1}^{m_1} v_i \cdot d_\infty(x, a^i)\right] + \left[\sum_{j=1}^{m_2} w_j \cdot d_\infty(x, b^j)\right] \to \min_{x \in \mathbb{R}^2},$$

where $v_1, \ldots, v_{m_1} \in \mathbb{R}$ are positive weights (e.g. demands of the desirable facilities) and $w_1, \ldots, w_{m_2} \in \mathbb{R}$ are negative weights (e.g. significance of the repulsion of the undesirable facilities, such as polluting factories or nuclear plants).

Summarizing, in our problem

$$1 \mid \mathrm{P} \mid v > 0, w < 0 \mid d_\infty \mid \text{median}$$

we search for one new facility (position 1: 1) in the plane (position 2: P), the given attraction facilities have positive weights (position 3: $v > 0$, i.e., $v_i > 0$ for all $i = 1, \ldots, m$), the given repulsion facilities have negative weights (position 3: $w < 0$, i.e., $w_i < 0$ for all $i = 1, \ldots, m_2$) and we consider a median problem (position 5: median), where we measure the distances between points using the maximum metric (position 4: d_∞).

2. Algorithm information and implementation

The corresponding algorithm included in the current version of the Software FLO generates exact solutions of the above the above location problem. The program uses an algorithm (Iterative Derivative Algorithm) that is a slight modification of the algorithm by Nickel and Dudenhöffer (1997) and a transformation between Manhattan norm and maximum norm.

The algorithm was implemented in FLO by Christian Günther. Software FLO has been able to solve the underlying location problem since program version 1.0.0, which was released on 22/04/2015.

3. Selected References

Further model and algorithm-specific information can be found in the following literature:

(A) S. Nickel and E. M. Dudenhöffer. *Weber's Problem with Attraction and Repulsion under Polyhedral Gauges.* Journal of Global Optimization, 11:409-432, 1997.

(B) A. Wagner. *A new Duality Based Approach for the Problem of Locating a Semi-Obnoxious Facility.* Dissertation, Martin Luther University Halle-Wittenberg, 2014.

(C) A. Wagner, J. E. Martinez-Legaz and Chr. Tammer. *Locating a Semi-Obnoxious Facility - A Toland-Singer Duality Based Approach.* Journal of Convex Analysis, 2015 (accepted).

(D) M. Hillmann. *Lagrange-Mulitplikatoren-Regeln und Algorithmen für nichtkonvexe Standortprobleme.* Masterarbeit, Martin-Luther-Universität Halle-Wittenberg, 2013.