Facility Location Optimizer

A tool for solving location problems.

Classification: $\mathbf{1} \mid \mathbf{P} \mid (+,-) \mid (d_1, \mu_j) \mid$ **Eff-vector**

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1. Problem formulation

The single-facility multiobjective location problem consists in minimizing the distances between a new facility $x \in \mathbb{R}^2$ and all given facilities

$$a^1 := (a_1^1, a_2^1),\ \cdots\ , a^m := (a_1^{m_1}, a_2^{m_1})\ \in\ \mathbb{R}^2$$

simultaneously. In addition to the attraction points $a^1, \ldots, a^{m_1}$, we now consider m_2 repulsion points

$$b^1 := (b_1^1, b_2^1),\ \cdots\ , b^{m_2} := (b_1^{m_2}, b_2^{m_2})\ \in\ \mathbb{R}^2,$$

(i.e., undesirable facilities, such as polluting factories or nuclear plants) and we want to maximize the distance from the new location point x to each of the points $b^1, \ldots, b^{m_2}$ simultaneously. Note that simultaneous minimization is understood in the sense of multiobjective optimization. Such problems are called

"Single facility multi-objective location problems
with attraction and repulsion in the plane"

in the literature of location theory. We measure the distances between the new facility and the given attraction facilities using the well-known Manhattan metric (also called rectangular metric or l_1 metric), which is defined by

$$d_1(x, a^i) := |x_1 - a_1^i| + |x_2 - a_2^i|$$

for all $x := (x_1, x_2) \in \mathbb{R}^2$ and all $a^i := (a_1^i, a_2^i)$, $i = 1, \dots, m_1$. Moreover, let the functions

$$\mu_1, \dots, \mu_{m_2} : \mathbb{R}^2 \to \mathbb{R}$$

represent gauge functions, i.e., we have

$$\mu_j(x - b^j) := \inf\{\lambda > 0 \mid x - b^j \in \lambda \cdot B_{\mu_j}\}$$

for all $j = 1, \dots, m_2$ and all $x \in \mathbb{R}^2$, where B_{μ_j} is a polytope with $0 \in \operatorname{int} B_{\mu_j}$ (in this case is μ_j a polyhedral gauge) or a unit ball of a l_p norm.

Now we are able to define our location problem through:

$$\begin{pmatrix} d_1(x, a^1) \\ \dots \\ d_1(x, a^{m_1}) \\ -\mu_1(x - b^1) \\ \dots \\ -\mu_{m_2}(x - b^{m_2}) \end{pmatrix} \to \operatorname*{v-min}_{x \in \mathbb{R}^2}.$$

Note that we search for Edgeworth-Pareto efficient solutions in the above-defined non-convex vector-valued location problem.

Summarizing, in our problem

$$1 \mid \mathrm{P} \mid (+, -) \mid (d_1, \mu_j) \mid \text{Eff-vector}$$

we search for one new facility (position 1: 1) in the plane (position 2: P), the given facilities are attraction points or repulsion points (position 3: $(+, -)$) and we consider a vector problem with regard to the solution concept of Edgeworth-Pareto efficiency (position 5: Eff-vector), where we measure the distances between the new facility and the attraction points using the Manhattan metric and the distances between the new facility and the repulsion points using specific gauge functions μ_j, $j = 1, \dots, m_2$ (position 4: (d_1, μ_j)).

2. Algorithm information and implementation

The corresponding algorithm included in the current version of the Software FLO solves the above-defined location problem and computes Edgeworth-Pareto efficient solutions using the following solution strategy (decomposition approach proposed by Alzorba, Günther and Popovici, 2015):

1°. Compute the set of all efficient solutions $\mathrm{Eff}(\mathbb{R}^2 \mid g)$ of the problem

$$g(x) := \begin{pmatrix} d_1(x, a^1) \\ \dots \\ d_1(x, a^{m_1}) \end{pmatrix} \to \underset{x \in \mathbb{R}^2}{\text{v-min}}\,.$$

2°. The decision maker has to select some negative weights $w_1, \dots, w_{m_2}$ depending on the significance of the repulsion of the undesirable facilities.

3°. Solve the scalar restricted location problem

$$h(x) := \sum_{j=1}^{m_2} w_j \cdot \mu_j(x - b^j) \to \min_{x \in \mathrm{Eff}(\mathbb{R}^2 | g)}\,.$$

4°. It holds that we have

$$x^0 \in \underset{x \in \mathrm{Eff}(\mathbb{R}^2 | g)}{\operatorname{argmin}}\, h(x) \subset \mathrm{Eff}(\mathbb{R}^2 \mid f) \cap \mathrm{Eff}(\mathbb{R}^2 \mid g).$$

Note that the current version of the Software FLO does not compute the whole set of Edgeworth-Pareto efficient solutions of the original underlying location problem. The Software computes solutions depending on the selection of the negative weights $w_1, \dots, w_{m_2}$ (e.g. significance of the repulsion of the undesirable facilities). More information about the model and the algorithm can be found in the paper by Alzorba, Günther and Popovici (2015).

The algorithm was implemented in FLO by Christian Günther. Software FLO has been able to solve the underlying location problem since program version 1.0.0, which was released on 22/04/2015.

3. Selected References

Further model and algorithm-specific information can be found in the following literature:

(A) S. Alzorba, C. Günther and N. Popovici. *A special class of extended multicriteria location problems.* Optimization, 64(5):1305-1320, 2015 (DOI: 10.1080/02331934.2013.869810).

(B) C. Günther. *Dekomposition mehrkriterieller Optimierungsprobleme und Anwendung bei nicht konvexen Standortproblemen.* Master-Thesis, Martin Luther University Halle-Wittenberg, 2013.

(C) A. Jourani, C. Michelot and M. Ndiaye. *Efficiency for continuous facility location problems with attraction and repulsion.* Annals OR, 167(1): 43-60, 2009.