Facility Location Optimizer

A tool for solving location problems.

Classification: $\mathbf{1} \mid \mathbf{P} \mid v > 0 \mid d_2 \mid$ **median**

1. Problem formulation

The goal is to find one new facility $x \in \mathbb{R}^2$ in the plane such that the weighted sum of the distances between the new facility x and given facilities located at the points $a^1, \ldots, a^m \in \mathbb{R}^2$ are minimized. Such problems are called

"Single facility median location problems in the plane"

in the literature of location theory and can be interpreted as a model for minimizing transportation costs. Using the well-known Euclidean metric, which is defined by

$$d_2(x, a^i) := \sqrt{(x_1 - a_1^i)^2 + (x_2 - a_2^i)^2}$$

for all $x := (x_1, x_2) \in \mathbb{R}^2$ and all $a^i := (a_1^i, a_2^i)$, $i = 1, \ldots, m$, the location problem is given by

$$\sum_{i=1}^{m} v_i \cdot d_2(x, a^i) = \sum_{i=1}^{m} v_i \cdot \sqrt{(x_1 - a_1^i)^2 + (x_2 - a_2^i)} \to \min_{x \in \mathbb{R}^2},$$

where $v_1, \ldots, v_m \in \mathbb{R}$ are positive weights (e.g. demands of the given facilities).

The above-defined location problem is known in the literature as the Fermat-Weber problem. Summarizing, in our problem

$$1 \mid \mathrm{P} \mid v > 0 \mid d_2 \mid \text{median}$$

we search for one new facility (position 1: 1) in the plane (position 2: P), the given facilities have positive weights (position 3: $v > 0$, i.e., $v_i > 0$ for all $i = 1, \dots, m$) and we consider a median problem (position 5: median), where we measure the distances between points using the Euclidean metric (position 4: d_2).

2. Algorithm information and implementation

The corresponding algorithm included in the current version of the Software FLO generates an approximative solution of the above location problem. The program uses the algorithm (Weiszfeld Algorithm) proposed in the paper by Weiszfeld (1937). At first, a special optimality criteria for the existing points (proposed by Kuhn, 1967) must be checked. After that the Weiszfeld iteration is performed as a fixed point iteration method. The user selects his preferred starting solution for the Weiszfeld iteration.

Note: A comprehensive and recently published overview over the Weiszfeld algorithm and its extensions (formulation and convergence results) is presented in the paper by Beck and Sabach (2015).

The algorithm was implemented in FLO by Christian Günther. Software FLO has been able to solve the underlying location problem since program version 1.0.0, which was released on 22/04/2015.

3. Selected References

Further model and algorithm-specific information can be found in the following literature:

(A) E. V. Weiszfeld. *Sur le point pour lequel la somme des distances de n points donnes est minimum.* Tohoku Math, 43:355386, 1937.

(B) R. F. Love, J. G. Morris and G. O. Wesolowsky. *Facility Location: Models and Methods.* North Holland, New York, 1988.

(C) H. W. Hamacher. *Mathematische Lösungsverfahren für planare Standortprobleme.* Vieweg Verlag, 1995.

(D) A. Beck and S. Sabach. *Weiszfelds Method: Old and New Results.* Journal of Optimization Theory and Applications, 164(1):1-40, 2015.