

Facility Location Optimizer

A tool for solving location problems.

Classification: **1** | **P** | $v > 0$ | d_2^2 | **median**

1. Problem formulation

The goal is to find one new facility $x \in \mathbb{R}^2$ in the plane such that the weighted sum of the distances between the new facility and given facilities located at the points $a^1, \dots, a^m \in \mathbb{R}^2$ are minimized. Such problems are called

“Single facility median location problems in the plane”

in the literature of location theory and can be interpreted as a model for minimizing transportation costs. Using the well-known squared Euclidean metric, which is defined by

$$d_2^2(x, a^i) := (x_1 - a_1^i)^2 + (x_2 - a_2^i)^2$$

for all $x := (x_1, x_2) \in \mathbb{R}^2$ and all $a^i := (a_1^i, a_2^i)$, $i = 1, \dots, m$, the location problem is given by

$$\sum_{i=1}^m v_i \cdot d_2^2(x, a^i) = \sum_{i=1}^m v_i \cdot ((x_1 - a_1^i)^2 + (x_2 - a_2^i)^2) \rightarrow \min_{x \in \mathbb{R}^2},$$

where $v_1, \dots, v_m \in \mathbb{R}$ are positive weights (e.g. demands of the given facilities).

Summarizing, in our problem

$$1 \mid P \mid v > 0 \mid d_2^2 \mid \text{median}$$

we search for one new facility (position 1: 1) in the plane (position 2: P), the given facilities have positive weights (position 3: $v > 0$, i.e., $v_i > 0$ for all $i = 1, \dots, m$) and we consider a median problem (position 5: median), where we measure the distances between points using the squared Euclidean metric (position 4: d_2^2).

2. Algorithm information and implementation

The corresponding algorithm included in the current version of the Software FLO generates the optimal solution of the above location problem. The program uses the algorithm (Center of Gravity Algorithm) proposed by White (1971).

The algorithm was implemented in FLO by Christian Günther. Software FLO has been able to solve the underlying location problem since version 1.0.0 (released on 22/04/2015).

3. Selected References

Further model and algorithm-specific information can be found in the following literature (only a small selection of references):

- (A) J. A. White. *A Quadratic Facility Location Problem*. AIIE Transactions, 3(2):156-157, 1971.
- (B) H. W. Hamacher. *Mathematische Lösungsverfahren für planare Standortprobleme*. Vieweg Verlag, 1995.