Facility Location Optimizer

A tool for solving location problems.

Classification: $\mathbf{1 \mid P \mid v > 0 \mid d_p \mid median}$

---

1. Problem formulation

The goal is to find one new facility $x \in \mathbb{R}^2$ in the plane such that the weighted sum of the distances between the new facility x and given facilities located at the points $a^1, \ldots, a^m \in \mathbb{R}^2$ are minimized. Such problems are called

"Single facility median location problems in the plane"

in the literature of location theory and can be interpreted as a model for minimizing transportation costs. Using the well-known l_p metric, which is defined by

$$d_p(x, a^i) := (|x_1 - a_1^i|^p + |x_2 - a_2^i|^p)^{\frac{1}{p}}$$

for all $x := (x_1, x_2) \in \mathbb{R}^2$ and all $a^i := (a_1^i, a_2^i)$, $i = 1, \ldots, m$, the location problem is given by

$$\sum_{i=1}^{m} v_i \cdot d_p(x, a^i) = \sum_{i=1}^{m} v_i \cdot (|x_1 - a_1^i|^p + |x_2 - a_2^i|^p)^{\frac{1}{p}} \to \min_{x \in \mathbb{R}^2},$$

where $v_1, \ldots, v_m \in \mathbb{R}$ are positive weights (e.g. demands of the given facilities).

Summarizing, in our problem

$$1 \mid \mathrm{P} \mid v > 0 \mid d_p \mid \text{median}$$

we search for one new facility (position 1: 1) in the plane (position 2: P), the given facilities have positive weights (position 3: $v > 0$, i.e., $v_i > 0$ for all $i = 1, \ldots, m$) and we consider a median problem (position 5: median), where we measure the distances between points using the l_p metric (position 4: d_p).

2. Algorithm information and implementation

The corresponding algorithm included in the current version of the Software FLO generates an approximative solution of the above location problem. The program uses the algorithm (Hyperbolic Approximation Algorithm) proposed in the paper by Morris and Verdini (1979). At first, a special optimality criteria for the existing points must be checked. After that the Hyperbolic Approximation iteration is performed as a fixed point iteration method. The user selects his preferred starting solution for the Hyperbolic Approximation iteration. More information about the procedure can be found in the book by Love, Morris and Wesolowsky (1988) or in the book by Hamacher (1995).

The algorithm was implemented in FLO by Christian Günther. Software FLO has been able to solve the underlying location problem since program version 1.0.0, which was released on 22/04/2015.

3. Selected References

Further model and algorithm-specific information can be found in the following literature:

(A) J. G. Morris and W. A. Verdini. *Minisum, l_p distance location problems solved via a perturbed problem and Weiszfeld's algorithm.* Operations Research, 27, 1180-1188, 1979.

(B) R. F. Love, J. G. Morris and G. O. Wesolowsky. *Facility Location: Models and Methods.* North Holland, New York, 1988.

(*C*) H. W. Hamacher. *Mathematische Lösungsverfahren für planare Standortprobleme.* Vieweg Verlag, 1995.