Facility Location Optimizer

A tool for solving location problems.

Classification: $\mathbf{1} \mid \mathbf{P} \mid v > 0,\ X = \bigcup_{i=1}^{q} P_i \mid \mu \mid$ **median**

1. Problem formulation

The feasible set X is given by a finite union of polytopes $P_1, \ldots, P_q \subseteq \mathbb{R}^2$ (i.e., bounded polyhedral sets in $\mathbb{R}^2$). Consequently, we have

$$X = \bigcup_{i=1}^{q} P_i.$$

The goal is to find one new facility $x \in X = \bigcup_{i=1}^{q} P_i$ such that the weighted sum of the distances between the new facility x and the given facilities located at the points $a^1, \ldots, a^m \in \mathbb{R}^2$ are minimized. Such problems are called

"Single facility median location problems
in the plane involving constraints"

in the literature of location theory and can be interpreted as a model for minimizing transportation costs.

Let B be a polytope (closed, bounded and polyhedral set) in $\mathbb{R}^2$ with $0 \in \operatorname{int} B$ and $B = -B$ (symmetry with respect to the origin). Moreover, assume that the polytope B can be represented by the convex hull of four points $r^1, r^2, r^3, r^4 \in \mathbb{R}^2$, i.e., it holds $B = \mathrm{conv}\{r^1, r^2, r^3, r^4\}$.

Now using the unit ball $B_\mu := B$ we can define a block norm μ. Note that the unit ball of the maximum norm (a special block norm) is determined by four extreme points

$$r^1 = (-1,1), r^2 = (1,1), r^3 = (1,-1), r^4 = (-1,-1) \in \mathbb{R}^2.$$

The distances induced by the block norm μ are defined by

$$\mu(x - a^i) := \inf\{\lambda > 0 \,|\, x - a^i \in \lambda \cdot B_\mu\}$$

for all $x := (x_1, x_2) \in \mathbb{R}^2$ and all $a^i := (a^i_1, a^i_2)$, $i = 1, \ldots, m$. Hence, the location problem is given by

$$\sum_{i=1}^{m} v_i \cdot \mu(x - a^i) \to \min_{x \in X = \bigcup_{i=1}^{q} P_i}$$

with $v_i > 0$ for all $i = 1, \ldots, m$ (e.g. demands of the given facilities).

Note: Constrained median location problems with positive weights and $q = 1$ (i.e., one polytope represents the feasible set) are convex, but for $q > 1$ these problems are non-convex in general (since a finite union of convex sets must not be convex in general).

Summarizing, in our problem

$$1 \mid \mathrm{P} \mid v > 0,\ X = \textstyle\bigcup_{i=1}^{q} P_i \mid \mu \mid \text{median}$$

we search for one new facility (position 1: 1) in the plane (position 2: P), the given facilities have positive weights, the search for the new facility is restricted to the feasible set $X = \bigcup_{i=1}^{q} P_i$ (position 3: $v > 0$, i.e., $v_i > 0$ for all $i = 1, \ldots, m$; $X = \bigcup_{i=1}^{q} P_i$) and we consider a median problem (position 5: median), where we measure the distances between points using a metric induced by a special class of block norms (position 4: μ).

2. Algorithm information and implementation

The corresponding algorithm included in the current version of the Software FLO generates the whole set of solutions of the above location problem. The program solves the location problem using an algorithm (Transformation Algorithm; see Reference A) that is based on the algorithm for solving the problem "1 | P | $v > 0$, $X = \bigcup_{i=1}^{q} P_i$ | d_1 | median" and an appropriate linear transformation.

Note: We assume that the set of solutions of the free problem "1 | P | $v > 0$ | μ | median" is completely contained in the interior of the polytope P such that no solution of the free problem is feasible for the restricted problem "1 | P | $v > 0$, $X = \bigcup_{i=1}^{q} P_i$ | μ | median".

The algorithm was implemented in FLO by Rico Khan during his Bachelor-Thesis (see Reference A). Software FLO has been able to solve the underlying location problem since version 1.2.0 which was released on 08/01/2016.

3. Selected References

Further model and algorithm-specific information can be found in the following literature:

(*A*) R. Khan. *Algorithmen für restringierte Standortprobleme einschließlich Implementierung*. Bachelor-Thesis, Martin Luther University Halle-Wittenberg, 2015.

(*B*) H. W. Hamacher. *Mathematische Lösungsverfahren für planare Standortprobleme*. Vieweg Verlag, 1995.