

# Facility Location Optimizer

A tool for solving location problems.

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**Classification:**  $1 \mid \mathbf{P} \mid v > 0, X = \bigcup_{i=1}^q P_i \mid d_2^2 \mid \text{median}$

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## 1. Problem formulation

The feasible set  $X$  is given by a finite union of polytopes  $P_1, \dots, P_q \subseteq \mathbb{R}^2$  (i.e., bounded polyhedral sets in  $\mathbb{R}^2$ ). Consequently, we have

$$X = \bigcup_{i=1}^q P_i.$$

The goal is to find one new facility  $x \in X = \bigcup_{i=1}^q P_i$  such that the weighted sum of the distances between the new facility  $x$  and the given facilities located at the points  $a^1, \dots, a^m \in \mathbb{R}^2$  are minimized. Such problems are called

“Single facility median location problems  
in the plane involving constraints”

in the literature of location theory and can be interpreted as a model for minimizing transportation costs.

Using the well-known squared Euclidean metric, which is defined by

$$d_2^2(x, a^i) := (x_1 - a_1^i)^2 + (x_2 - a_2^i)^2$$

for all  $x := (x_1, x_2) \in \mathbb{R}^2$  and all  $a^i := (a_1^i, a_2^i)$ ,  $i = 1, \dots, m$ , the location problem is given by

$$\sum_{i=1}^m v_i \cdot d_2^2(x, a^i) = \sum_{i=1}^m v_i \cdot ((x_1 - a_1^i)^2 + (x_2 - a_2^i)^2) \rightarrow \min_{x \in X = \bigcup_{i=1}^q P_i}$$

with positive weights  $v_1, \dots, v_m \in \mathbb{R}$  (e.g. demands of the given facilities).

*Note:* Constrained median location problems with positive weights and  $q = 1$  (i.e., one polytope represents the feasible set) are convex, but for  $q > 1$  these problems are non-convex in general (since a finite union of convex sets must not be convex in general).

Summarizing, in our problem

$$1 \mid P \mid v > 0, X = \bigcup_{i=1}^q P_i \mid d_2^2 \mid \text{median}$$

we search for one new facility (position 1: 1) in the plane (position 2: P), the given facilities have positive weights, the search for the new facility is restricted to the feasible set  $X = \bigcup_{i=1}^q P_i$  (position 3:  $v > 0$ , i.e.,  $v_i > 0$  for all  $i = 1, \dots, m$ ;  $X = \bigcup_{i=1}^q P_i$ ) and we consider a median problem (position 5: median), where we measure the distances between points using the squared Euclidean metric (position 4:  $d_2^2$ ).

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## 2. Algorithm information and implementation

The corresponding algorithm included in the current version of the Software FLO generates the whole set of solutions of the above location problem. The program uses a modified and extended version of the algorithm (Projection Algorithm) formulated in the book by Hamacher (1995).

The algorithm was implemented in FLO by Rico Khan during his Bachelor-Thesis (see Reference A). Software FLO has been able to solve the underlying location problem since version 1.2.0 which was released on 08/01/2016.

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## 3. Selected References

Further model and algorithm-specific information can be found in the following literature:

- (A) R. Khan. *Algorithmen für restringierte Standortprobleme einschließlich Implementierung*. Bachelor-Thesis, Martin Luther University Halle-Wittenberg, 2015.
- (B) H. W. Hamacher. *Mathematische Lösungsverfahren für planare Standortprobleme*. Vieweg Verlag, 1995.