Stopping and bending light in 2D photonic structures

Alejandro B. Aceves and Tomáš Dohnal

Department of Mathematics, University of New Mexico,
Albuquerque, NM 87131, USA
aceves@math.unm.edu, dohnal@math.unm.edu

Abstract: We present the problem of optical pulses interacting with localized defects in 2D photonic structures, where the grating in the direction of propagation is assumed to be in Bragg resonance with the electric field.

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1 Uniform grating structures

We consider a waveguide geometry with confinement of light in one transverse direction (y). The refractive index is periodic in both the direction of propagation (z) and the other transverse direction (x). Furthermore, the periodicity in the direction of propagation (z) is assumed to be in Bragg resonance with the wavelength of the electric field, thus creating strong back reflection.

The regime of wave propagation through these gratings that we are interested in is when the coupling between forward and backward propagating modes is of the order of the nonlinear length. A coupled system for the slowly varying envelopes of the electric field can be derived from Maxwell's Equations. The derivation is typically done using a multiple scales expansion under the assumption that the characteristic length scales of the coupling, the diffraction and the nonlinearity are in balance. The dynamics in uniform fiber gratings are then governed by the two dimensional Coupled Mode Equations with a nonnegative periodic $x-$potential (for the 1D case see for example chapter 2 of [5])

$$\begin{aligned} i(\partial_t + c_g\partial_z)E_+ + \partial^2_{x^2}E_+ + \kappa E_- + P(x)E_+ + \Gamma(|E_+|^2 + 2|E_-|^2)E_+ = 0 \\ i(\partial_t - c_g\partial_z)E_- + \partial^2_{x^2}E_- + \kappa E_+ + P(x)E_- + \Gamma(|E_-|^2 + 2|E_+|^2)E_- = 0, \end{aligned} \tag{1}$$

where E_+, E_- denote the forward and backward propagating envelopes respectively; and without any loss of generality $c_g, \kappa, \Gamma > 0$. We typically take $P(x) = \delta(1 + \cos(\nu x))$ with $\delta, \nu \geq 0$.

In the 1D case, for fiber gratings, where the confinement of light is in both x and y directions (equation (1) without the diffraction term $\partial^2_{x^2}E_\pm$ and the potential $P(x)$) stable pulse propagation is possible thanks to the existence of gap solitons ([2], [3]). These are solitary waves with frequency inside the forbidden gap of the linear system ($\Gamma = 0$) and can propagate with velocities ranging between 0 and the speed of light in the corresponding uniform medium. The main advantage of fiber gratings as compared to bare fibers, from application point of view, is the distance at which solutions converge to stable pulses. In bare fibers it is hundreds of meters whereas in fiber gratings it is within centimeters.

In the 2D case without the $x-$grating (equation (1) without $P(x)$) no stable pulse propagation is possible due to the diffraction. This can be argued directly from the Hamiltonian structure of the system and has been seen both in the reduced variational analysis of [1] and in our numerical experiments, where all so far studied localized pulses rapidly diffract in the x direction. Since our objective, as introduced in the next section, is to model interaction of pulses with localized defects, stable pulse propagation is crucial for the study.

We propose that by introducing a grating in x, i.e. the periodic potential $P(x)$, stable pulse propagation occurs as demonstrated in Figure 1. We will also investigate whether a specific $z-$profile in the initial condition of (1) yields both E_+ and E_- propagating together in the same direction (referring to the 1D case where for gap solitons E_+, E_- do co-propagate).

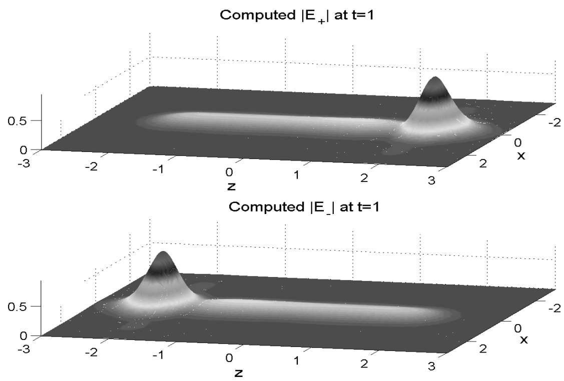

Fig. 1. Solution to (1) with $c_g = 2, \kappa = 1, \Gamma = 1, \delta = 10, \nu = 2$

2 Periodic structures with defects

We introduce localized defects (local variations to the refractive index) to the above described medium and study whether pulses can be stopped or whether the direction of their propagation can be changed due to the interaction with these defects. Potential engineering applications of this are in optical memory, rerouting of pulses or all-optical switches.

An analogous study has been done in [4] in 1D for fiber gratings. There, the authors studied how gap solitons interact with carefully selected defects. The governing model is

$$\begin{aligned} i(\partial_t + c_g\partial_z)E_+ + \kappa(z)E_- + V(z)E_+ + \Gamma(|E_+|^2 + 2|E_-|^2)E_+ = 0 \\ i(\partial_t - c_g\partial_z)E_- + \kappa(z)E_+ + V(z)E_- + \Gamma(|E_-|^2 + 2|E_+|^2)E_- = 0. \end{aligned} \tag{2}$$

They showed that for a given defect with its defect mode (stationary solution of (2) of the form $e^{-i\omega t}\mathcal{E}_\pm(z)$) with frequency ω and total intensity I they had to launch a pulse of the same frequency (resonance) and equal or higher total intensity (energetic accessibility) to achieve strong interaction.

In our 2D case we select defects that can be represented as a sum of 1D defects: $V_1(x) + V_2(z)$. This is in order to be able to use a separation of variables ansatz $E_\pm(x,z,t) = G(x)F_\pm(z,t)$ for finding the linear defect modes. The governing linear system is

$$\begin{aligned} i(\partial_t + c_g\partial_z)E_+ + \kappa(z)E_- + \partial^2_{x^2}E_+ + (V_1(x) + V_2(z) + P(x))E_+ = 0 \\ i(\partial_t - c_g\partial_z)E_- + \kappa(z)E_+ + \partial^2_{x^2}E_- + (V_1(x) + V_2(z) + P(x))E_- = 0 \end{aligned} \tag{3}$$

and after separating the variables

$$[i\partial_t + ic_g\sigma_3\partial_z + V_2(z) - \mu + \kappa(z)\sigma_1]\,\vec{F} = 0 \tag{4a}$$

$$G'' + (V_1(x) + P(x) + \mu)G = 0, \tag{4b}$$

with $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \vec{F} = \begin{pmatrix} F_+ \\ F_- \end{pmatrix}$ and μ the separation constant.

First we solve the eigenvalue problem (4b) numerically with $V_1(x) = \beta\text{sech}(\eta x)$ (with the condition of a localized eigenfunction with no zeros) to obtain (μ, G). Then, using the same ansatz and the same $z-$defect $V_2(z)$ as in [4], we solve (4a). An example solution is

$$\kappa(z) = e^{i\alpha}\left[(\omega-\mu)^2 + k^2\tanh^2\left(\frac{k}{c_g}z\right)\right]^{1/2}, \quad \alpha \in \{0,\pi\}$$

$$V_2(z) = \frac{1}{2}\frac{k^2(\omega-\mu)\text{sech}^2\left(\frac{k}{c_g}z\right)}{(\omega-\mu)^2 + k^2\tanh^2\left(\frac{k}{c_g}z\right)}.$$

$$\vec{F} = e^{-i\omega t} \begin{pmatrix} \exp\left(\frac{i}{2}\arctan\left(\frac{k\tanh\left(\frac{k}{c_g}z\right)}{\omega-\mu}\right)\right) \\ ie^{-i\phi}\exp\left(\frac{-i}{2}\arctan\left(\frac{k\tanh\left(\frac{k}{c_g}z\right)}{\omega-\mu}\right)\right) \end{pmatrix} \operatorname{sech}\left(\frac{k}{c_g}z\right).$$

The plot of the modulus $|E_+|$ for the above linear defect mode at time $t = 1$ is shown in Figure 2. The solution is stationary with its modulus constant in time. The parameter choice is $\beta = 5, \eta = 10, \delta = 10, \nu = 2, \omega = \mu + 1, k = 4, \phi = \frac{3}{2}\pi$ and $c_g = 1$. The corresponding eigenvalue of (4b) is $\mu \approx -16.96$. The plotted solution was obtained integrating (3) using the Discontinuous Galerkin method with the initial condition $\begin{pmatrix} E_+ \\ E_- \end{pmatrix}(x, z, t = 0) = G(x)\vec{F}(z, t = 0)$ as given above.

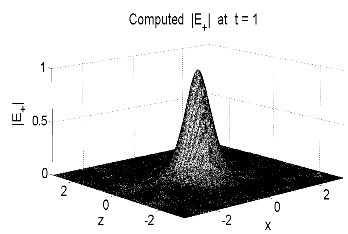

Fig. 2. The modulus of a linear defect mode at time $t = 1$

Using the linear defect modes, an example of which was given above we perform a perturbative construction of nonlinear defect modes of small total intensity and for large intensity numerically solve the full nonlinear eigenvalue problem

$$\begin{aligned} (\omega + ic_g\partial_z)E_+ + \kappa(z)E_- + \partial^2_{x^2}E_+ + (V_1(x) + V_2(z) + P(x))E_+ \\ +\Gamma(|E_+|^2 + 2|E_-|^2)E_+ = 0 \\ (\omega - ic_g\partial_z)E_- + \kappa(z)E_+ + \partial^2_{x^2}E_- + (V_1(x) + V_2(z) + P(x))E_- \\ +\Gamma(|E_-|^2 + 2|E_+|^2)E_- = 0. \end{aligned} \tag{5}$$

Finally, we investigate the possibility of trapping pulses on these defects as well as changing direction of their propagation due to the contact with a defect.

References

1. A.B.Aceves, B.Constantini and C.De Angelis, "Two-dimensional gap solitons in a nonlinear periodic slab waveguide," Journ. of the Opt. Soc. Am B **12**, 1475 (1995).
2. W.Chen and D.L.Mills, "Gap solitons and the nonlinear optical response of superlattices," Phys. Rev. Lett. **58**, 160 (1987).
3. B.J.Eggleton, R.E.Slusher, C.M.de Sterke, P.A.Krug and J.E.Sipe, "Bragg grating solitons," Phys. Rev. Lett. **76**, 1627 (1996).
4. R.H.Goodman, R.E.Slusher and M.I.Weinstein, "Stopping light on a defect," Journ. of the Opt. Soc. Am B **19**, 1635 (2002).
5. R.E.Slusher, B.J.Eggleton, "Nonlinear Photonic Crystals," Springer-Verlag, New York, 2003.