Dynamics driven by dispersive shocks in the strong nonlinear regime of optics

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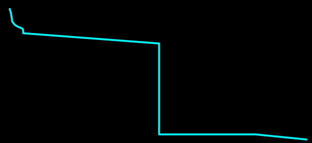
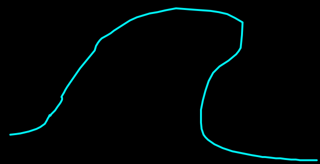
Outline

- shock waves : effect of dispersion (vs. dissipation)
- envelope shocks in the semiclassical 1+1(2)D NLS
- spatial: shocks surviving nonlocality (thermal lensing)
- temporal: shocks with periodic input (fwm)
- carrier shock (Maxwell-Lorentz): a challenge for NLO
- summary

Shock waves

Shock wave: a (moving) discontinuity appearing
in the field (inviscid shock in hyperbolic PDEs)



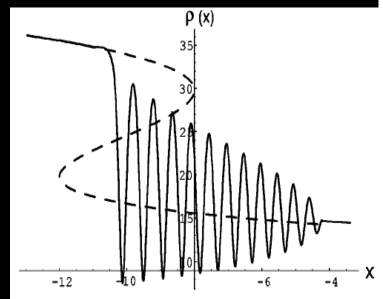


common phenomenon in different areas:
plasmas, superfluids, gas dynamics, BEC, water waves

also appear in NL-PDE dispersive models

hydrodynamics with photons

in dispersion (vs dissipation) dominated systems,
shocks lead to fast oscillations
dispersive (collisionless) shock wave = undular bore

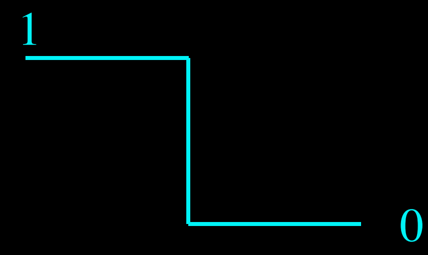


[Gurevich & Pitaevskii, JETP (1973); Kamchatnov, 2000]

Shock in dissipative system

Flux conservative equation

$$u_z + f_x(u) = 0 \;\; ; \;\; f(u) = \frac{u^2(x,z)}{2}$$



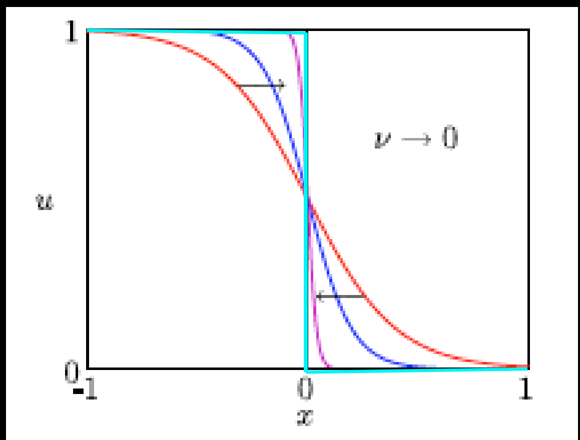
Discontinuous solutions are regularized by dissipation

Burger equation

$$u_z + \left(\frac{u^2}{2}\right)_x = \nu u_{xx}$$

(smooth) TW shock

$$u(z,x) = \frac{1}{2}\left\{1 + \tanh\left[\frac{1}{4\nu}\left(\frac{z}{2} - x\right)\right]\right\}$$

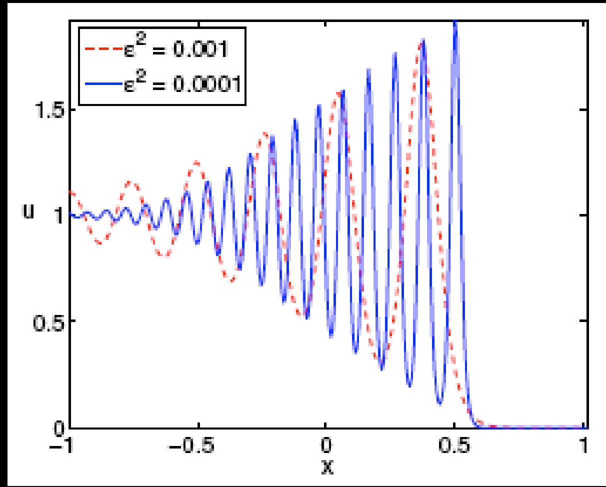


velocity = 1/2 regardless of dissipation

dispersive regularization (universal models)

KdV $u_z + \left(\frac{u^2}{2}\right)_x = -\varepsilon^2 u_{xxx}$

small disp added to flux conserved eq



Zabusky & Kruskal, Interaction of "solitons" in a collisionless plasma…, PRL (1965)

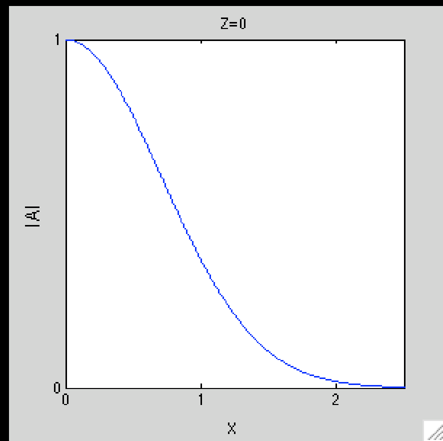
standard NLO model: weakly disp. NLS

$$i\frac{\partial A}{\partial Z} - \frac{k''}{2}\frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A = 0$$

temporal

$$i\frac{\partial A}{\partial Z} + \frac{1}{2k}\nabla_\perp^2 A - \gamma f(|A|^2)A = 0$$

spatial (paraxial diffraction)



non-soliton (wave-breaking) regime

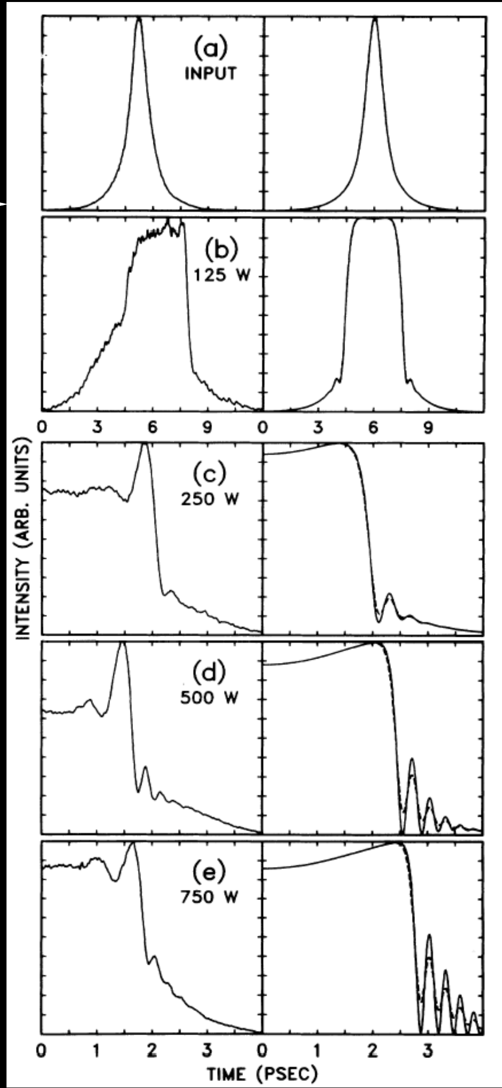
- NL & disp add up (defocusing)
- NL dominates dispersion

wave-breaking-free beams (parabolic) similaritons

experimental evidence in NLO

G-P oscillations manifest as wave-breaking of pulses in fibers (NLS eq with normal GVD)

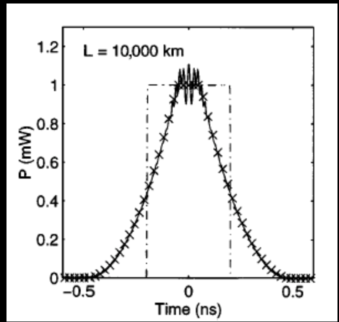
Grischowsky & Rothenberg, PRL (1989)



shocking is quite sensitive to input condition

NRZ (square) pulses do not shock

Kodama & Wabnitz, OL (1995)



math-oriented approach:
Forest, Kutz, McLaughlin, 1998; Kodama, 1999

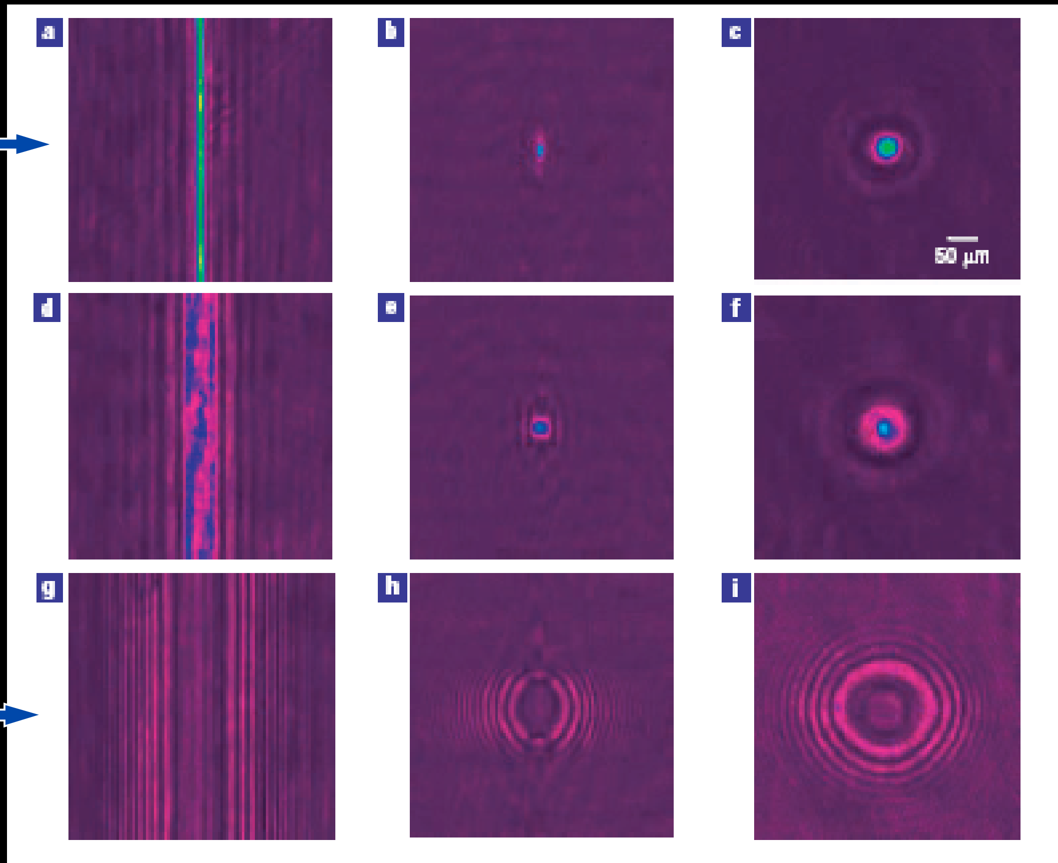
what about beams (spatial shock) ?

Gaussian beams on background

1D (stripe) elliptical 2D gaussian

input

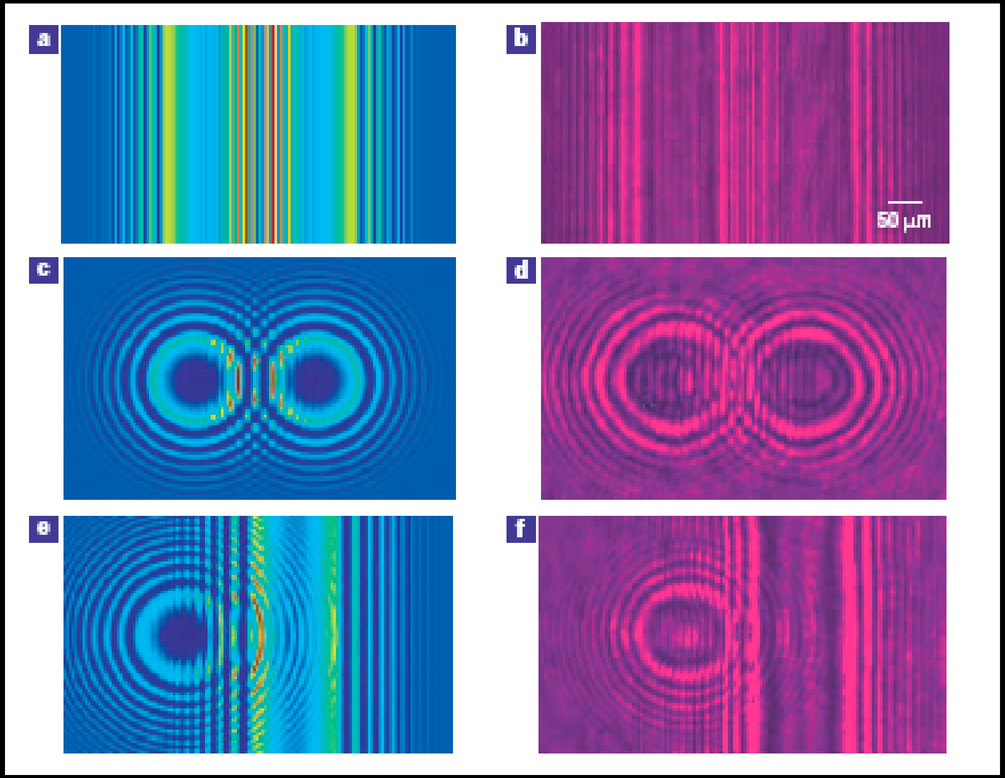
output



Rings with screening photorefractive NL (defocusing)

W. Wan, S. Jia, J. W. Fleischer, Nature Phys., 2006

Interactions between dispersive shocks



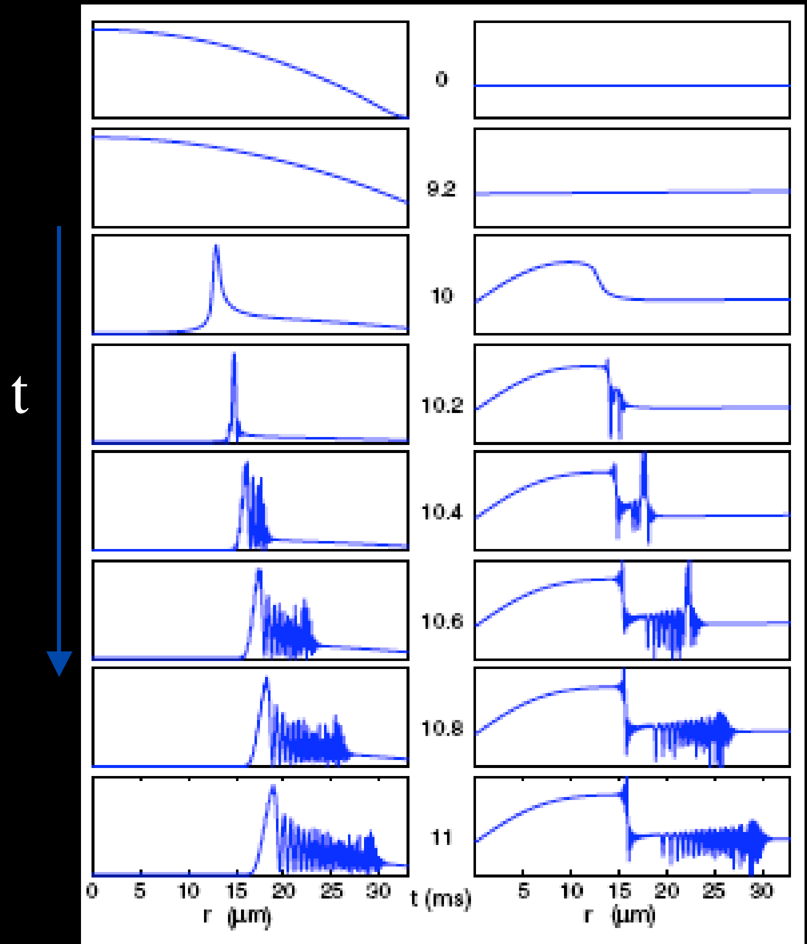
interaction of collisionless shocks is not trivial
(nonlinear superposition)

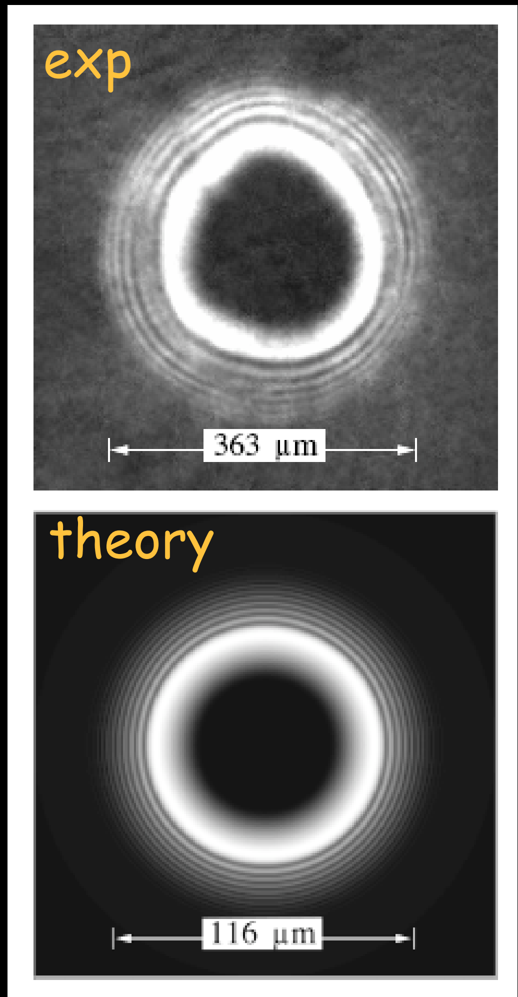
W. Wan, S. Jia, J. W. Fleischer, Nature Phys., 2006

Bose-Einstein Condensation

mean-field NLS eq - kinetic spreading=diffraction

atom density   radial velocity   "blast" waves





blaster:
laser +
antitrap

Hoefer et al., Phys. Rev. A, 2006

The semiclassical limit (NLS)

$$L \; ; \; L_{nl} \equiv \frac{1}{\gamma I_{peak}} \; ; \; L_d \equiv 2kw_0^2$$

sample — nonlinear — diffraction

3 length scales involved

$$i\,\varepsilon \frac{\partial \psi}{\partial z} + \frac{\delta}{2}\,\varepsilon^2\,\nabla_\perp^2 \psi - |\psi|^2\,\psi = 0$$

$$z = \frac{Z}{L} \; ; \; x, y = \frac{X, Y}{w_0} \; ; \; \psi = \frac{A(X,Y,Z)}{\sqrt{I_{peak}}} \; ; \; \varepsilon = \frac{L_{nl}}{L} \; ; \; \delta\varepsilon^2 = \frac{L_{nl}}{L_d}$$

$$i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \psi = V(\psi)\psi$$

$$V(\psi) = |\psi|^2$$

$$\epsilon \ll 1 \qquad \delta \sim 1$$

$$L_{nl} \ll L \qquad \sqrt{L_{nl} L_d} \simeq L$$

$$\Longleftrightarrow$$

$$L_{nl} \ll L \ll L_{disp}$$

weakly dispersive - strongly nonlinear regime !!!

WKB approach

ansatz $$\psi(z,x)=\sqrt{\rho(z,x)}\exp\left[i\frac{1}{\varepsilon}s(z,x)\right]$$ yields (leading order)

1D shallow-water (SW) wave hyperbolic (flux cons.) eqs.

$$u_z + uu_x + \rho_x = 0$$ continuity

$$\rho_z + (\rho u)_x = 0$$ Eulero

⟺

Riemann

$$\partial_t \begin{pmatrix} r^+ \\ r^- \end{pmatrix} = -\begin{pmatrix} u+\sqrt{h} & 0 \\ 0 & u-\sqrt{h} \end{pmatrix} \partial_x \begin{pmatrix} r^+ \\ r^- \end{pmatrix}$$

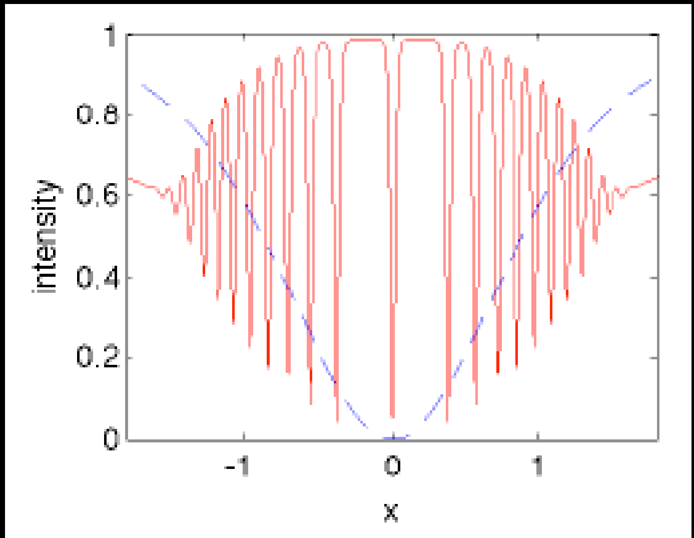
eigenvelocities

	ρ (h)	u=ds/dx	x	z
photons (us)	power	chirp	tranv (time)	distance
water waves	height	velocity	space	time
gas dynamics ($p \sim \rho^2$)	density	velocity	space	time

SW -> shock for proper initial (z=0) data (Lax criterion)

beyond wave-breaking point ?

the oscillating dispersive shock wave is a modulated periodic solution of the dispersive NL-PDE



Asymptotic (t ~ 1/ε, ε<<1) **NLS, KdV**

Whitham approach :

periodic TW solution (cn-oidal)
with slowly-varying parameters

$$\frac{\partial r_i}{\partial z} + v_i(r)\frac{\partial r_i}{\partial x} = 0; \quad i = 1, 2, 3, 4$$

search for simple wave solutions of Whitham eqs

useful formulation in the framework of inverse scattering method

e.g. Kamchatnov, NL periodic waves and their modulations

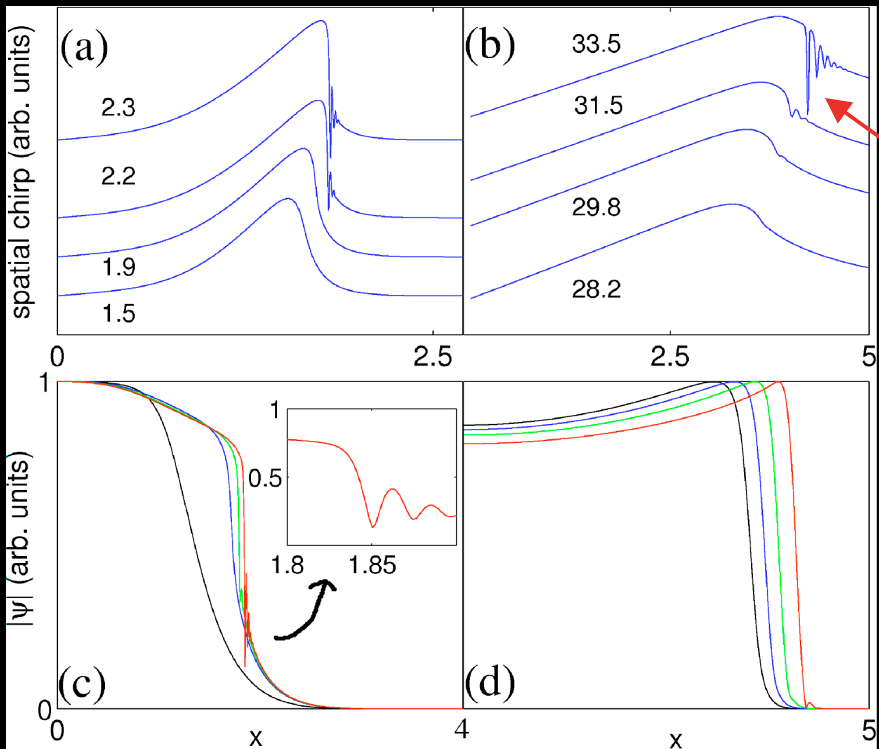
Effect of nonlocality

Nonlocal response (transport, finite-range, diffusion)

average NL around points (reorientational, liquid crystals)

Expect nonlocality to smooth out shock phenomena

local (NLS) nonlocal



Gaussian input

nonlocal dynamics :
driven by phase (u)
power front following

PRL, in press

A general nonlocal model

$$i\,\varepsilon\frac{\partial\psi}{\partial z}+\frac{\varepsilon^2}{2}\nabla_\perp^2\psi+\chi\,\delta n\,\psi=-i\frac{\alpha}{2}\,\varepsilon\psi,$$
$$-\sigma^2\nabla_\perp^2\delta n+\delta n=|\psi|^2,$$

$$z=\frac{Z}{L}\;;\;x,y=\frac{X,Y}{w_0}\;;\;\psi=\frac{A(X,Y,Z)}{I_{peak}}\;;\;\delta n=k_0L_{nl}\Delta n(X,Y,Z)\;;\;\alpha=\alpha_0 L$$
$$\varepsilon=\frac{L_{nl}}{L}=\sqrt{\frac{L_{nl}}{L_d}}\;;\;L\equiv\sqrt{L_{nl}L_d}$$

perform WKB reduction

1D

$$\rho_z+(\rho u)_x=0$$
$$u_z+uu_x-\chi\delta n_x=0$$
$$-\sigma^2\delta n_{xx}+\delta n=\rho$$

2D

$$\rho_z+\left[(\rho u)_r+\frac{1}{r}\rho u\right]=0$$
$$u_z+uu_r-\chi\delta n_r=0$$
$$-\sigma^2\left(\delta n_{rr}+\frac{1}{r}\delta n_r\right)+\delta n=\rho$$

does gaussian beams shock despite nonlocality ??

An approximate analytical solution

Solve for dn neglecting changes in $\rho(z)$

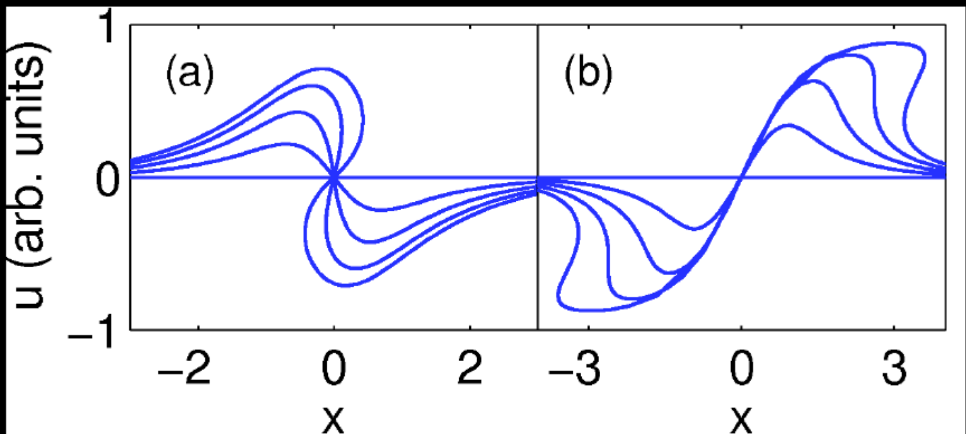
semi-linear eq.for the phase $$P(x,z,u)u_z + Q(x,z,u)u_x = R(x,z,u)$$

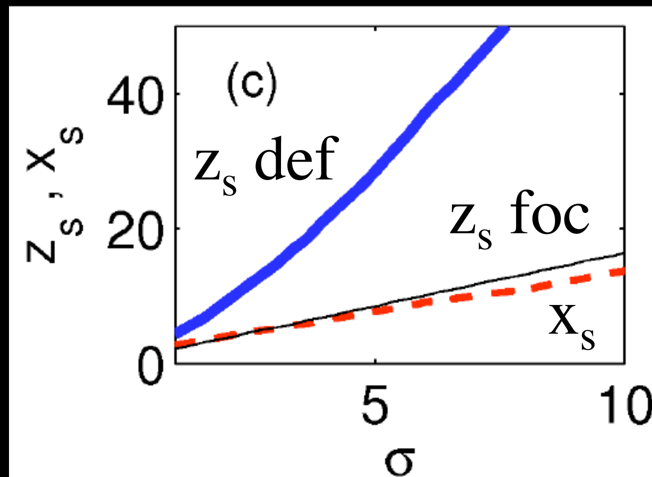
theory of characteristics
parametric form $$\frac{dx}{dz} = u \; ; \; \frac{du}{dz} = \chi \, \delta n_x$$

u(x) @ different z → shock

focusing defocusing

shock position z_s x_s
vs nonloc degree σ



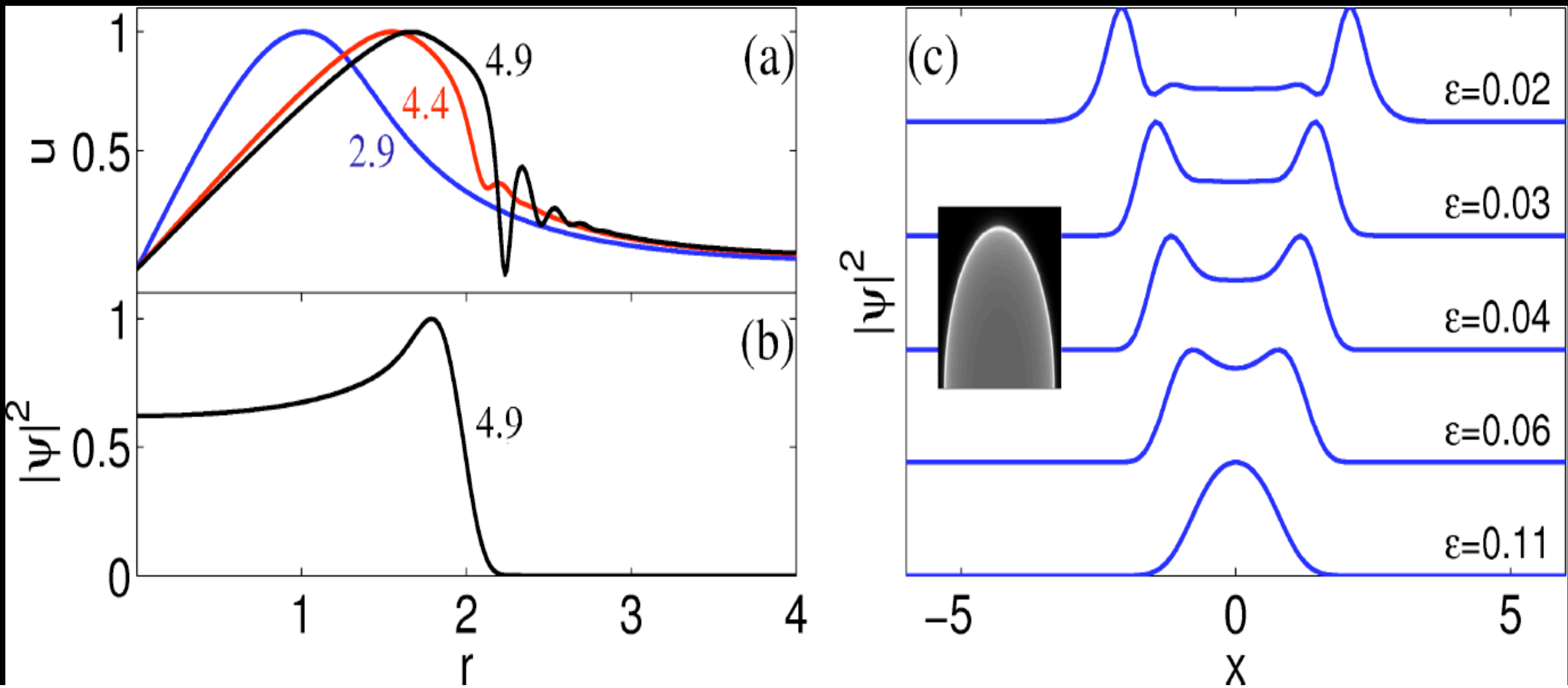


defocusing regime: numerics

solving numerically the full NL nonlocal model
(paraxial with loss + NL nonlocal dn)

2D (round) shock in u
(phase) moving outward
power front following

exp : fixed length sample
change power (L_{nl}) - smallness ε



Thermal nonlinearities

Light absorbed ➡ heating ➡ refr. index change ➡ thermal lens

Gordon et al., J. Appl. Phys. (1965)

steady-state

$$\Delta n = \frac{dn}{dT}\delta T \quad \Leftrightarrow \quad \nabla^2 \delta T(X,Y,Z) = -\gamma |A|^2$$

$$\gamma = \frac{\alpha_0}{\rho_0 c_p D_T}$$

attenuation constant

density, specific heat, diffusion

dn/dT<0 defocusing - divergent lens - thermal blooming

self-induced lens is not aberration-free

usually treated as uncoupled to e.m. wave eq.

heat eq. ruling also (thermal) soliton regime

z-dependence of T ignored - infinite range nonlocality

C. Rotschild, O. Cohen, O. Manela, M. Segev and T. Carmon, PRL 95, 213904 (2005)

Thermal blooming - our model

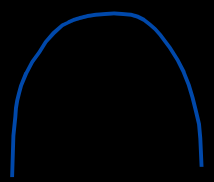
heat equation

$$(\partial_X^2 + \partial_Y^2)\delta T_\perp(X,Y) - C\delta T_\perp(X,Y) = -\gamma|A|^2$$

Phenomenological: A. Yakimenko et al., PRE 71, 065603(R) (2005)

longitudinal T-profile

parabolic approx



$$\delta T(X,Y,Z) = \left[1 - \frac{(Z-\hat{Z})^2}{2L_{eff}^2}\right]\delta T_\perp(X,Y)$$

$$\nabla^2\delta T \simeq (\partial_X^2 + \partial_Y^2)\delta T_\perp - \frac{\delta T_\perp}{L_{eff}^2} \quad \Leftrightarrow \quad C = \frac{1}{L_{eff}}$$

normalization

$$\delta n = k_0 L_{nl}\left|\frac{dn}{dT}\right|\delta T_\perp \quad ; \quad \sigma^2 = \frac{1}{Cw_0^2} = \frac{L_{eff}^2}{w_0^2}$$

nonlocal model

recovered

$$-\sigma^2\nabla_\perp^2\delta n + \delta n = |\psi|^2$$

$$i\,\varepsilon\frac{\partial\psi}{\partial z} + \frac{\varepsilon^2}{2}\nabla_\perp^2\psi - \delta n\;\psi = -i\frac{\alpha}{2}\varepsilon\;\psi$$

Thermal blooming - numbers

two limits

thin sample - link L_{eff} to material param.

$$L_{eff} \to \infty$$

∞-range nonlocal model recovered

$$\text{small } L_{eff} \quad \Leftrightarrow \quad |(\partial_X^2 + \partial_Y^2)\delta T_\perp| << \frac{|\delta T_\perp|}{L_{eff}^2}$$

$$L_{eff} = \sqrt{\frac{|n_2|}{\gamma |dn/dT|}} = \sqrt{\frac{D_T \rho_0 c_p |n_2|}{\alpha_0 |dn/dT|}}$$

Rhodamine B water solution

Z-scan data : $\alpha = 62\ cm^{-1}$; $n_2 = 7\ 10^{-7}\ cm^2\ W^{-1}$

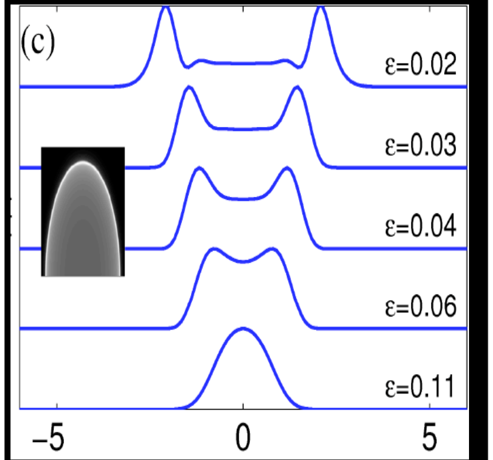
$dn/dT = 10^{-4}\ K^{-1}$ $\gamma = 10^4\ K\ W^{-1}$ → $L_{eff} = 20\ \mu m$

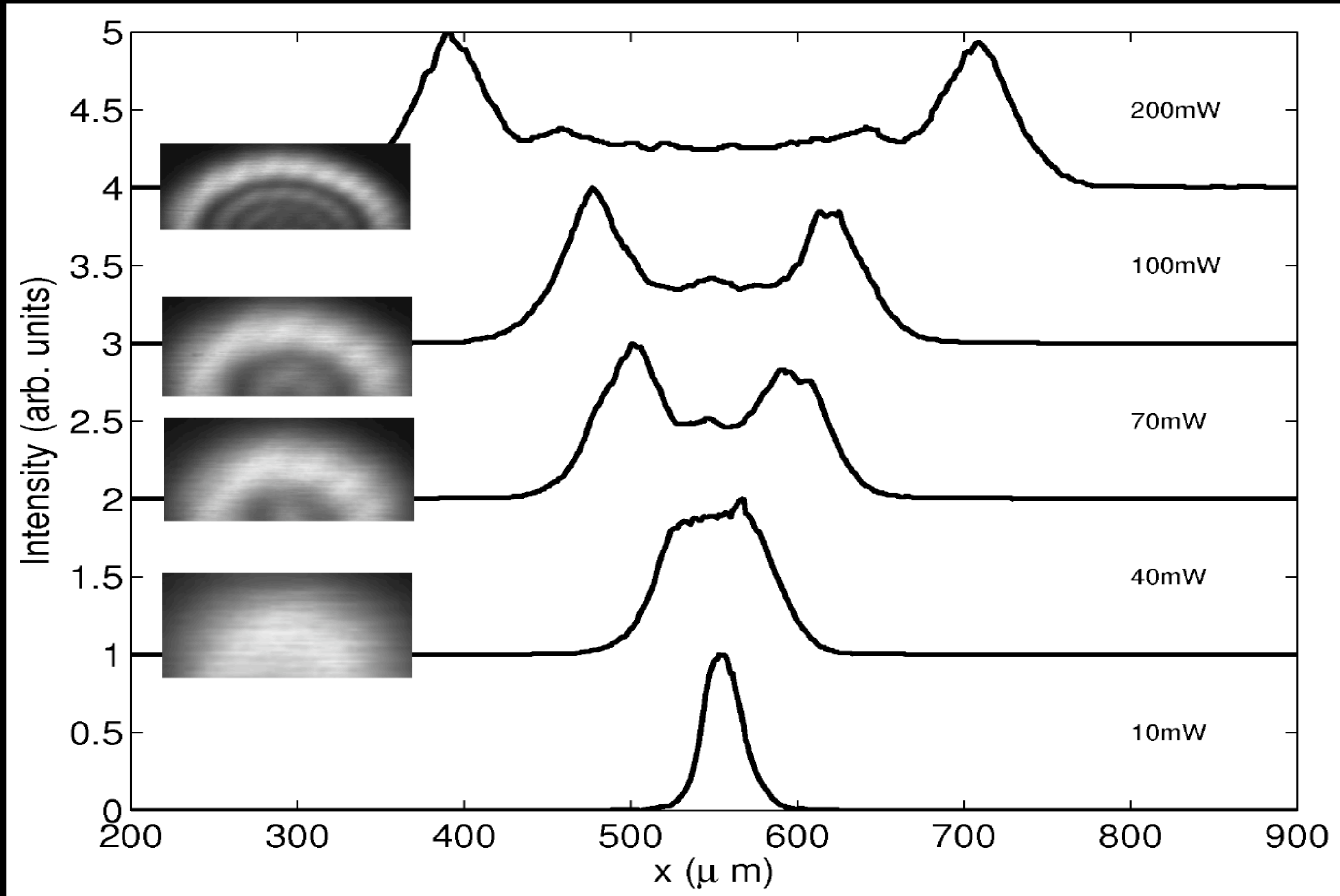
$\sigma = 0.3$

$\varepsilon = 0.025$ (P=200 mW)

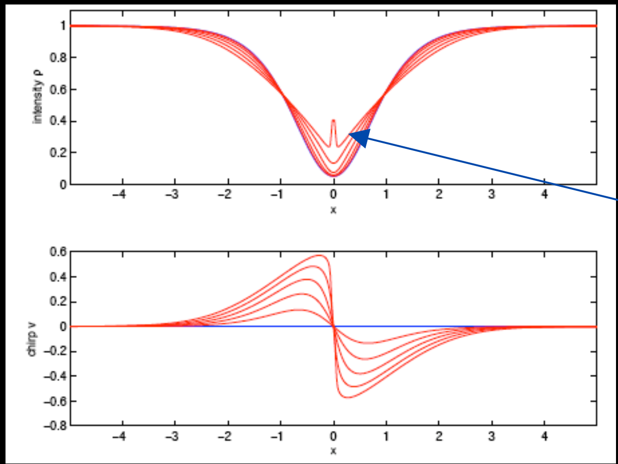
Spatial experiment : self-defocusing

Rhodamine B in water (L=1 mm cell), waist w_0= 20 μm





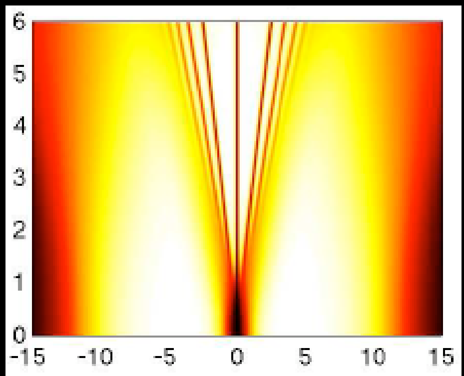
disp. shock from a single dark soliton

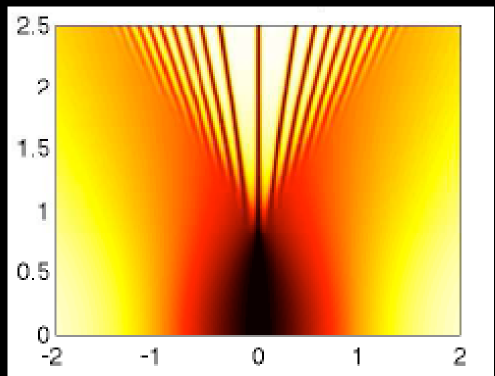


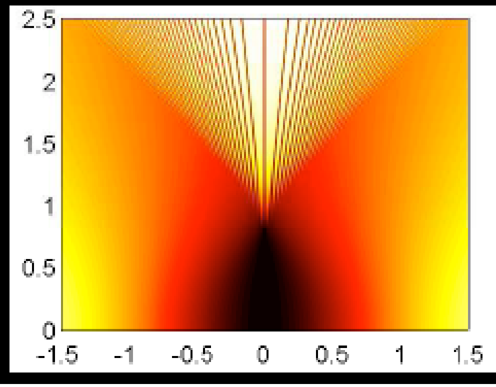
input: dark soliton (ε= 1)

WKB limit of local NLS : 2 fronts

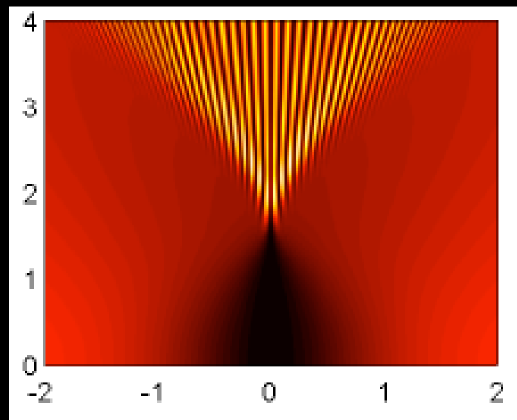
local NLS decreasing ε







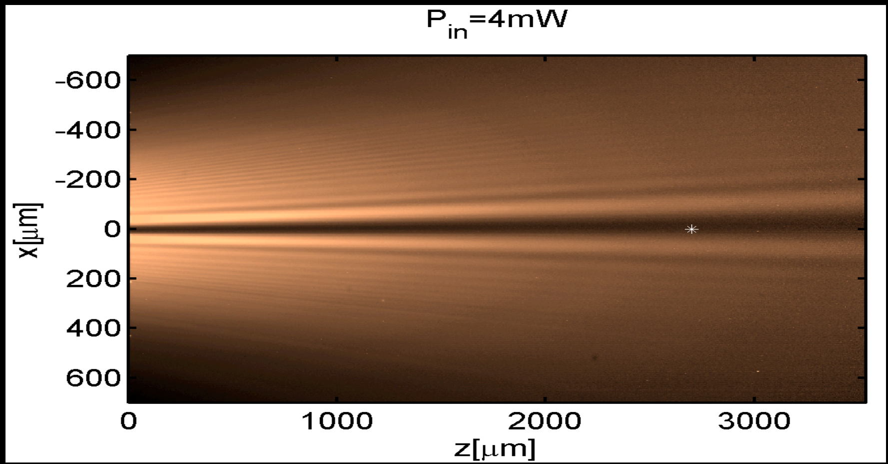
nonlocal NLS
(similar)



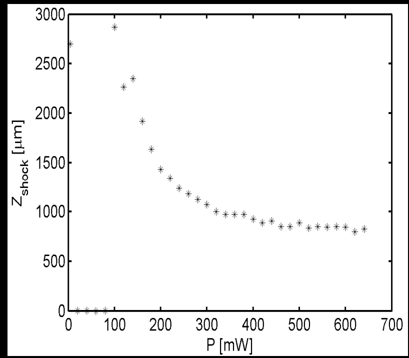
see also Kamchatnov, PRE 2002

Experiment (dark soliton input)

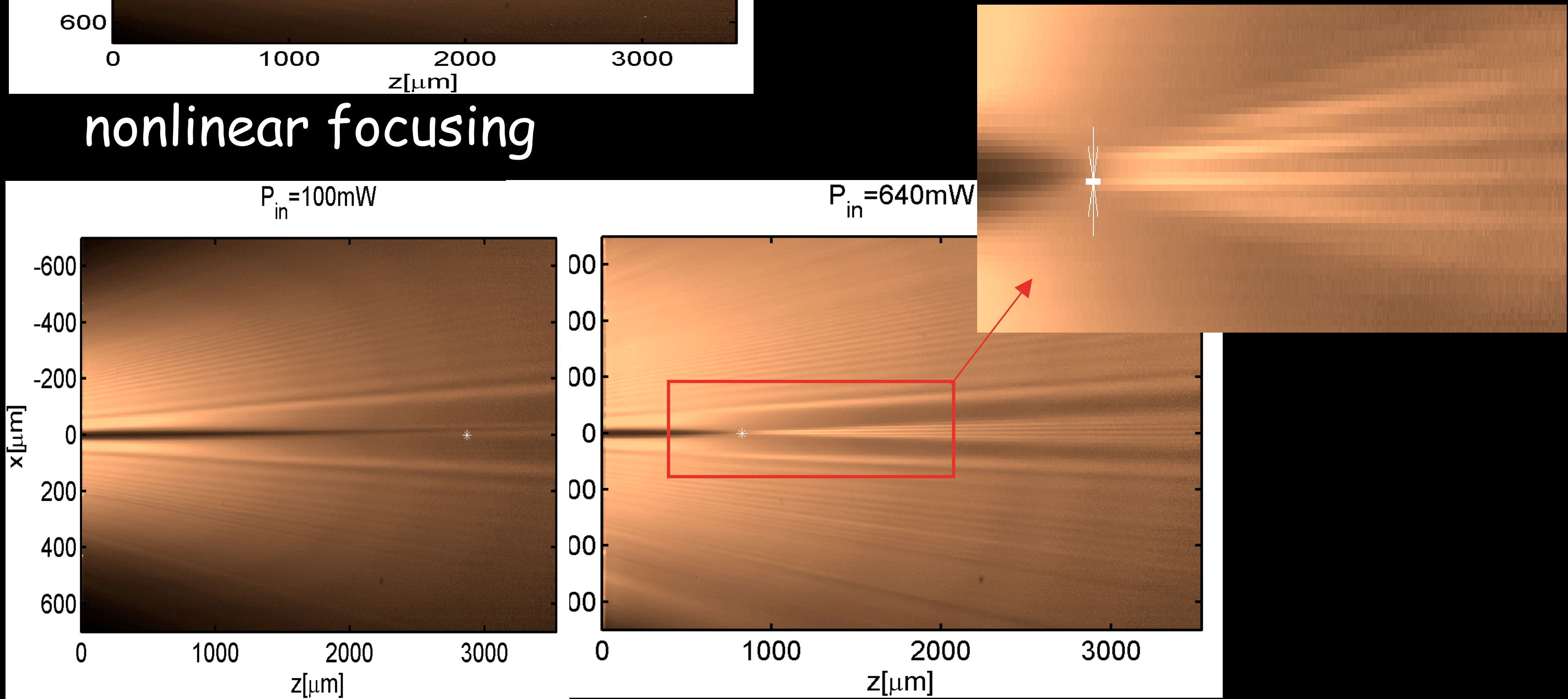
linear (low power)



shock distance vs power



nonlinear focusing

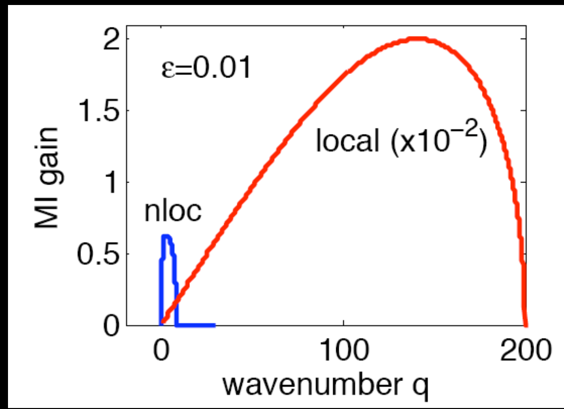


the focusing (MI) regime

local NLS: WKB non-hyp, ill-posed (eigenvel. Im) [Miller,Bronski,…]

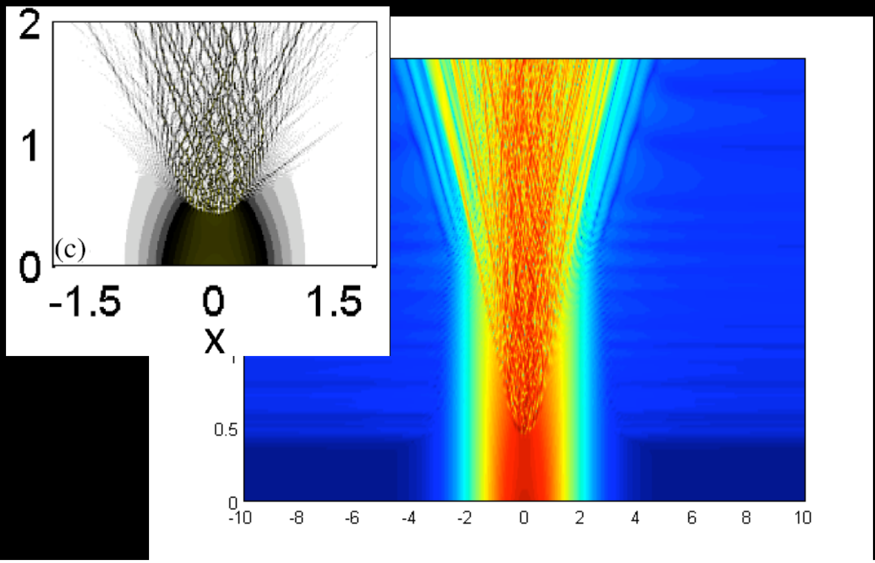
large bandwidth, high gain Modulation Instability

$$g_{MI} = \frac{1}{\varepsilon}\sqrt{\frac{\varepsilon^2 q^2}{2}\left(\frac{2\chi}{1+\sigma^2 q^2} - \frac{\varepsilon^2 q^2}{2}\right)}$$

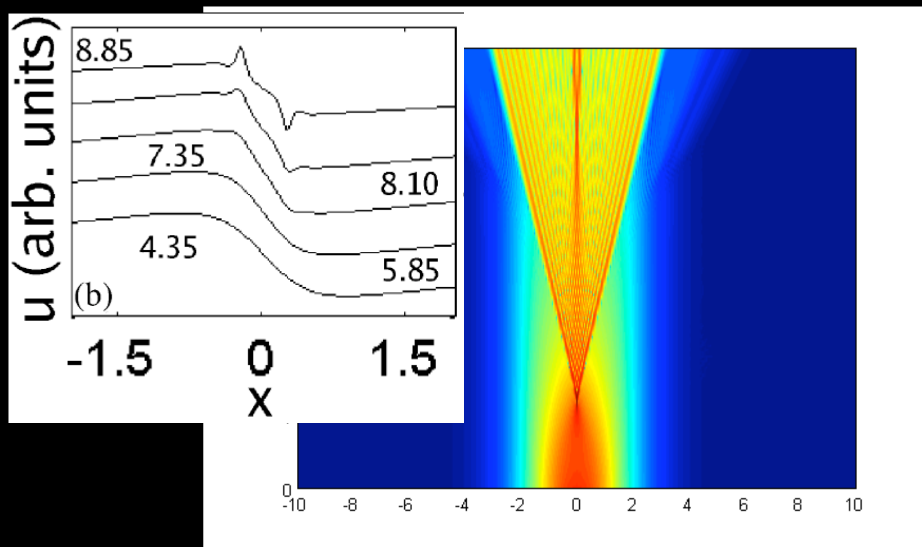


nonlocality suppresses MI

quasi-local

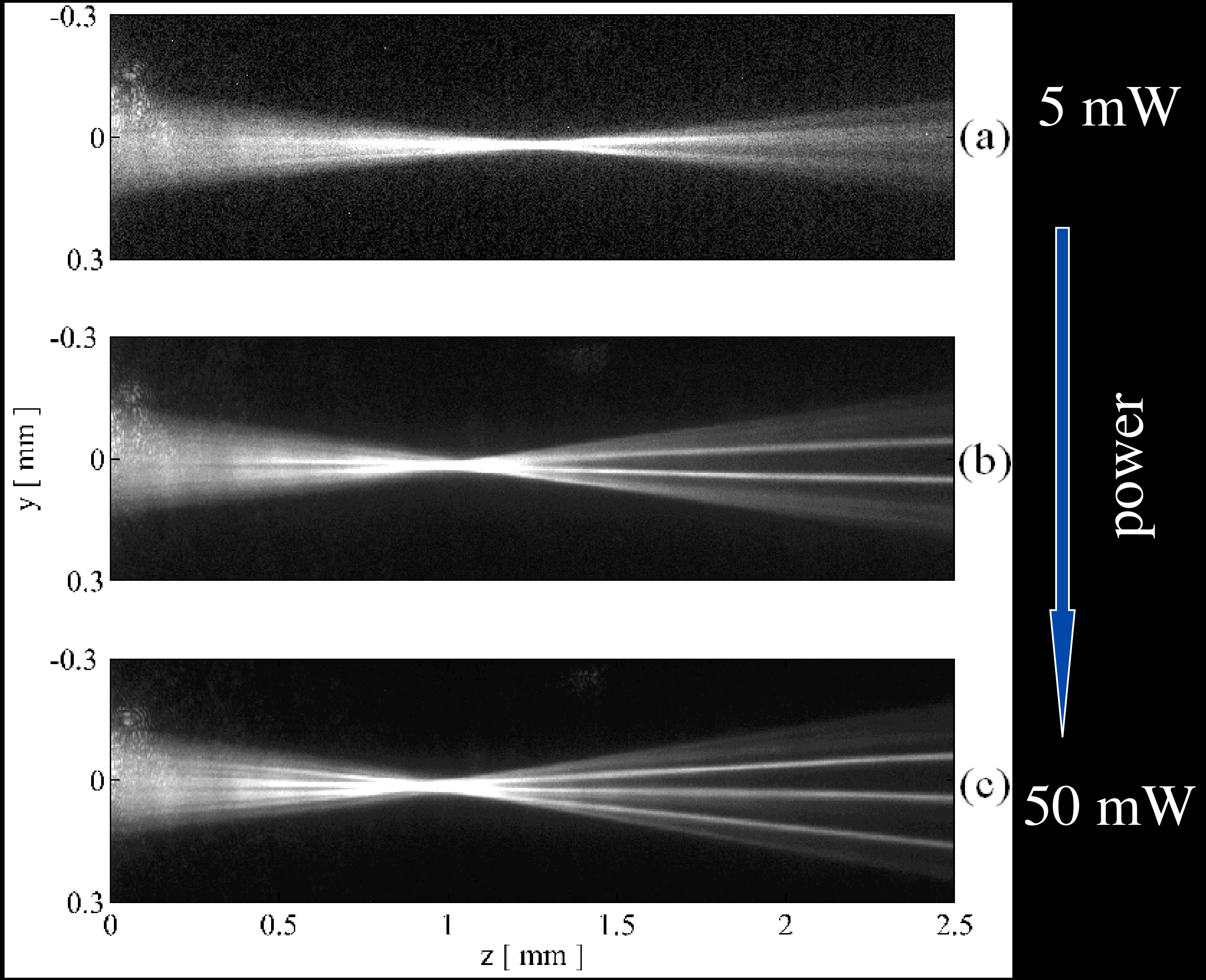


nonlocal (shock driven)



Spatial experiment : self-focusing

Is there any evidence for self-focusing media ?



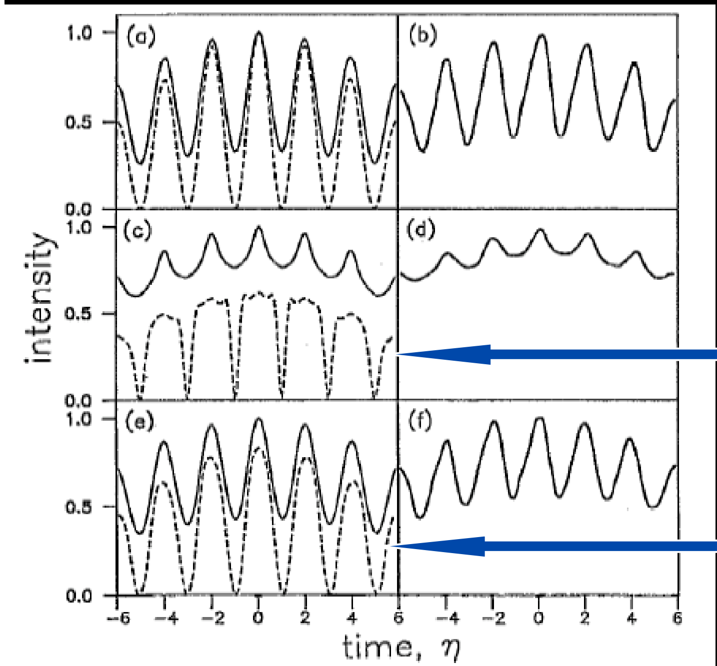
bright filaments

nematic liquid crystals

(molecular reorientation nonlinearity)

Peccianti, Conti, Assanto, OL 28, 2231 (2003)

back to local: periodic input (fwm)

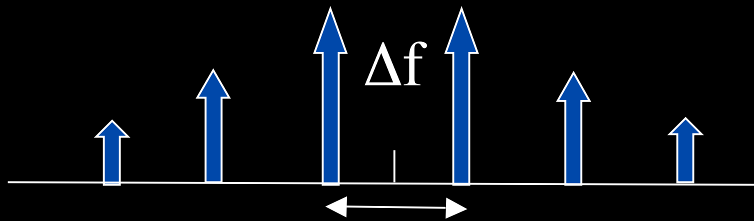


experiment, 1994

temporal (normal GVD)

Δf= 150-600 GHz   L ≈ 1 Km

Mamyshev et al. (CREOL) APL (1994)

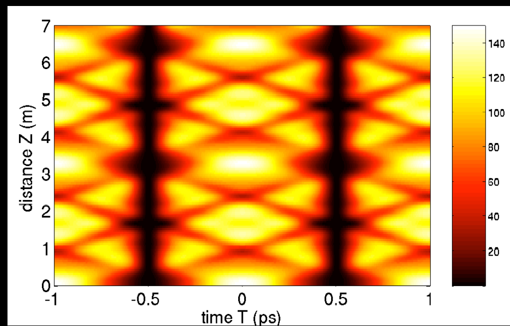


compression

dark solitons

four-wave mixing

restoration (recurrence)



numerics

At low frequency Δf enter semiclasical regime

$\varepsilon \sim \Delta f \,/\, \mathrm{Power}^{1/2}$

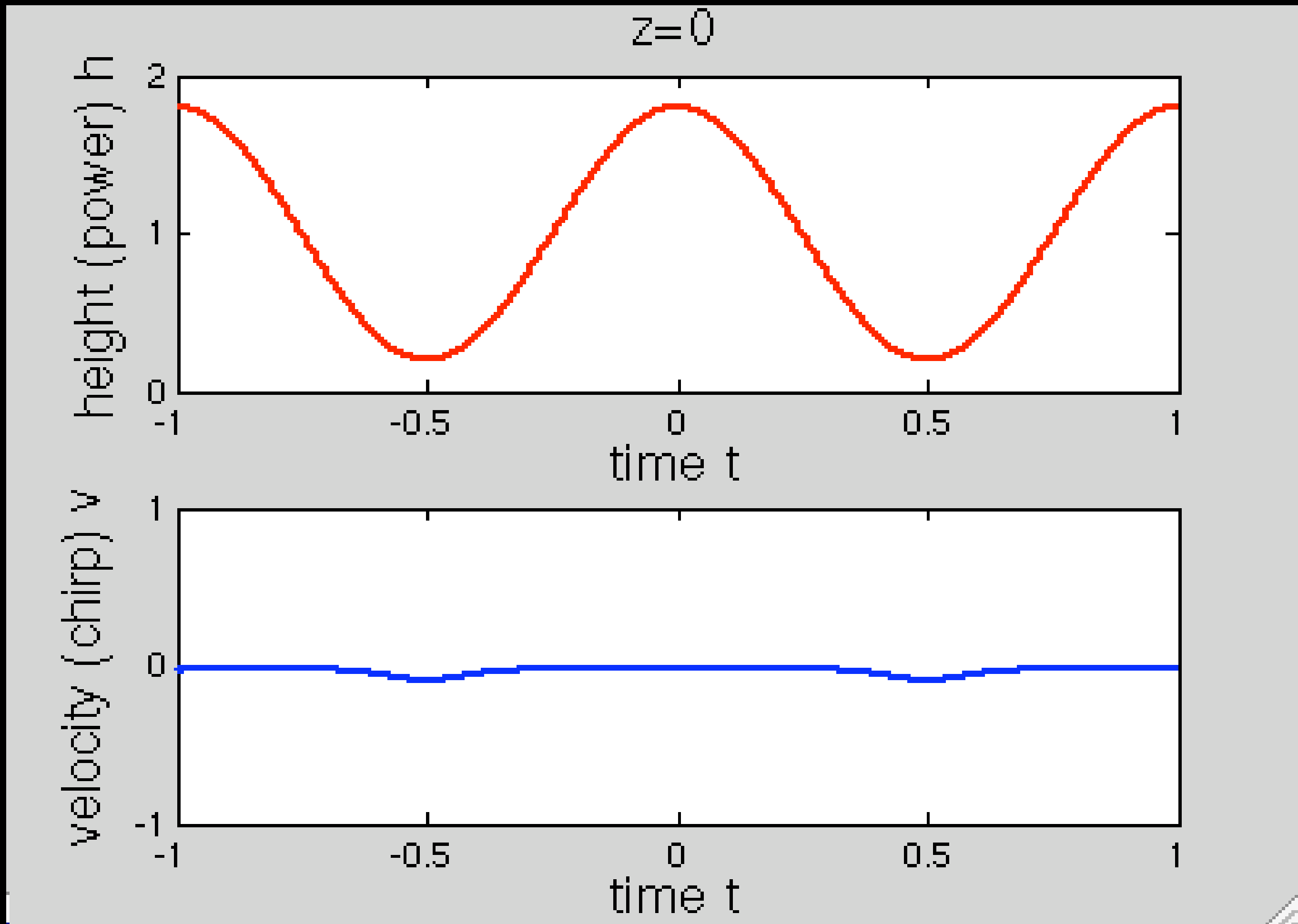
dispersive shock array ➡ colliding dispersive shocks

snapshots of semiclassical NLS dynamics

2-lines (periodic) input

$$\psi(z=0,x) = \sqrt{\eta}\exp(ik_x x) + \sqrt{1-\eta}\exp(-ik_x x)$$

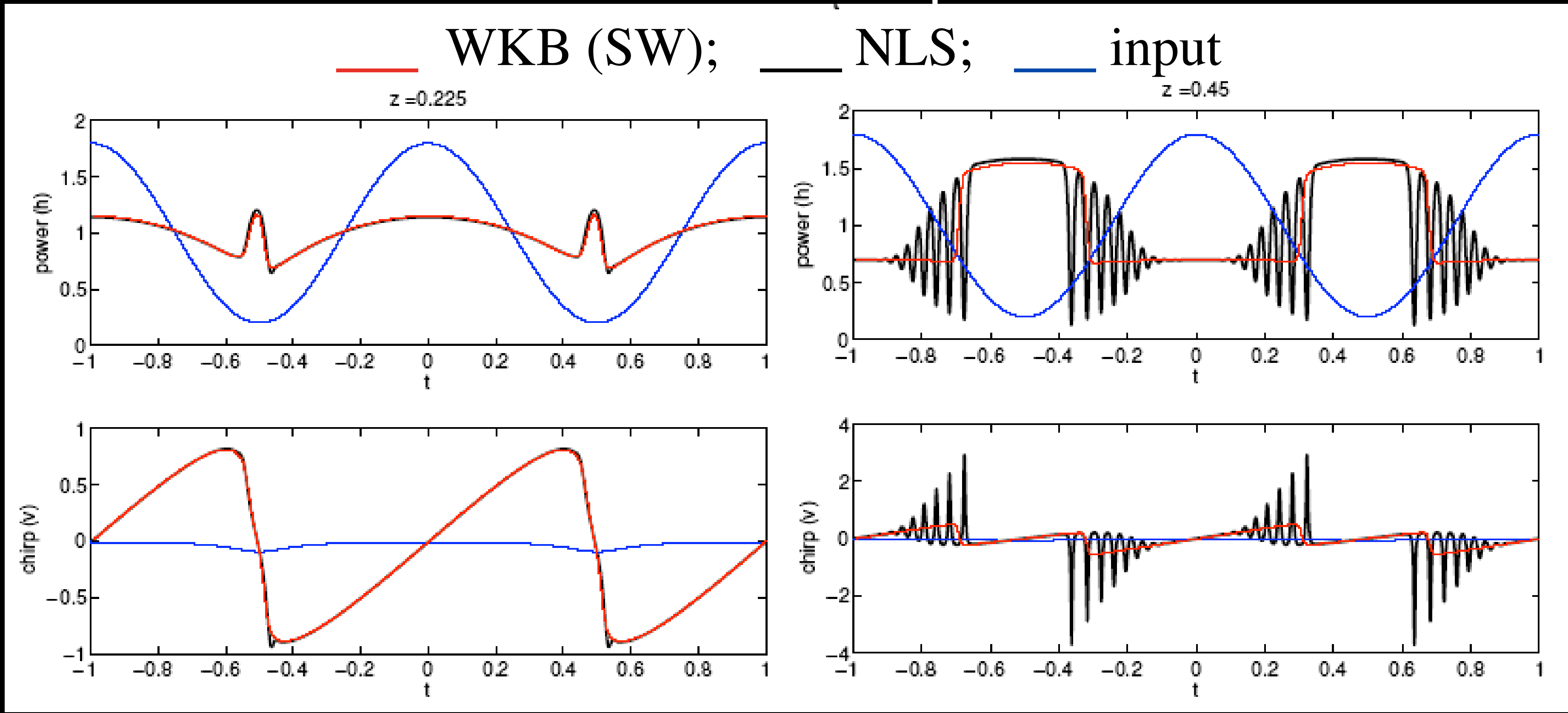
$$\psi(z=0,t) = \sqrt{\eta}\exp(i\omega t/2) + \sqrt{1-\eta}\exp(-i\omega t/2)$$



η=0.8
ε=0.01

WKB (shallow-water) vs NLS model

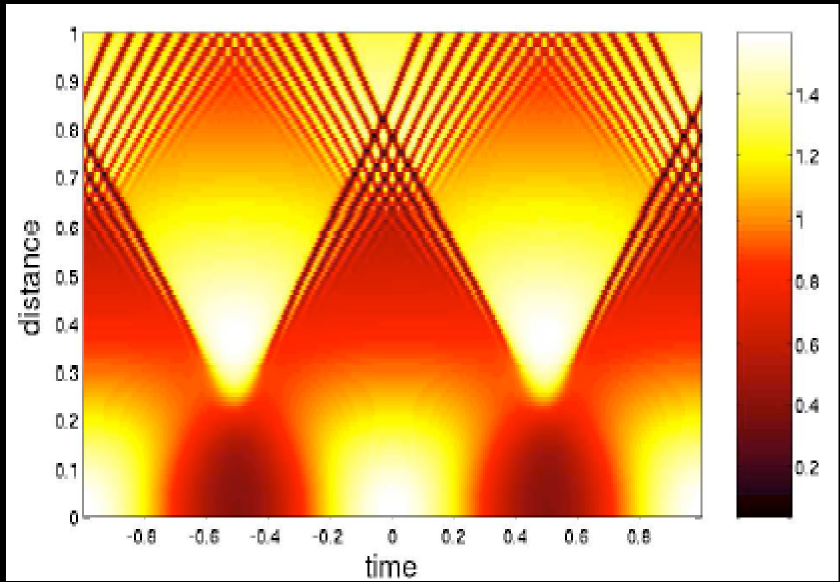
close to (right) shock point   post-shock oscillations



post-shock oscillations behave individually as dark solitons

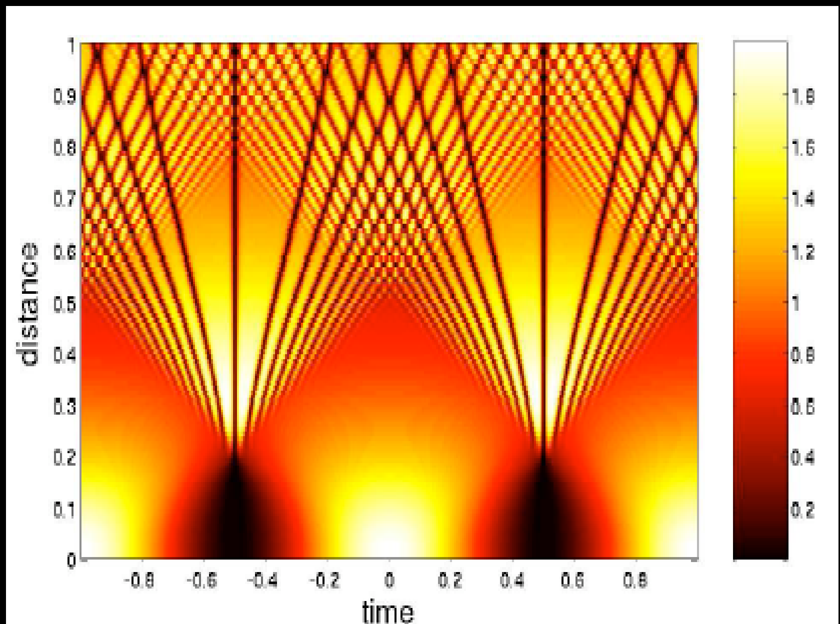
- velocity depends on darkness (the darker the slower)
- survive collisions
- right phase jump

multi-“soliton” collisions



imbalance η=0.8

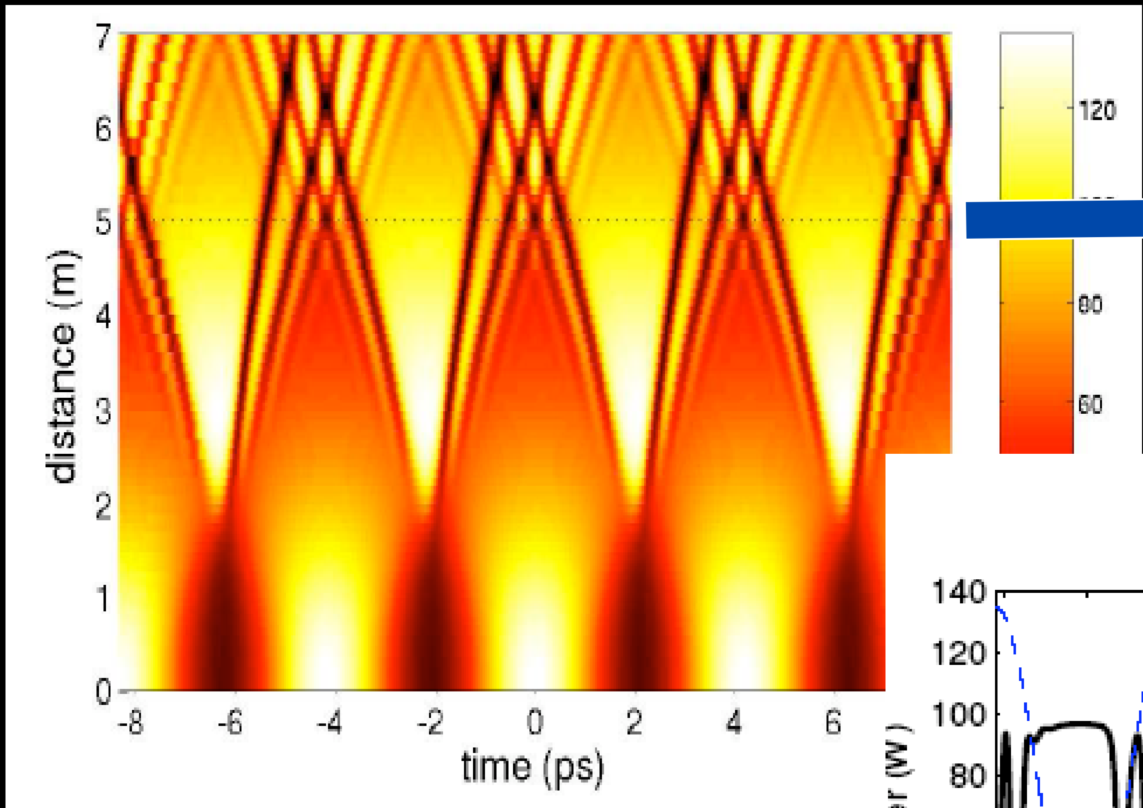
of “solitons” increases as ε decreases



balanced η=0.5

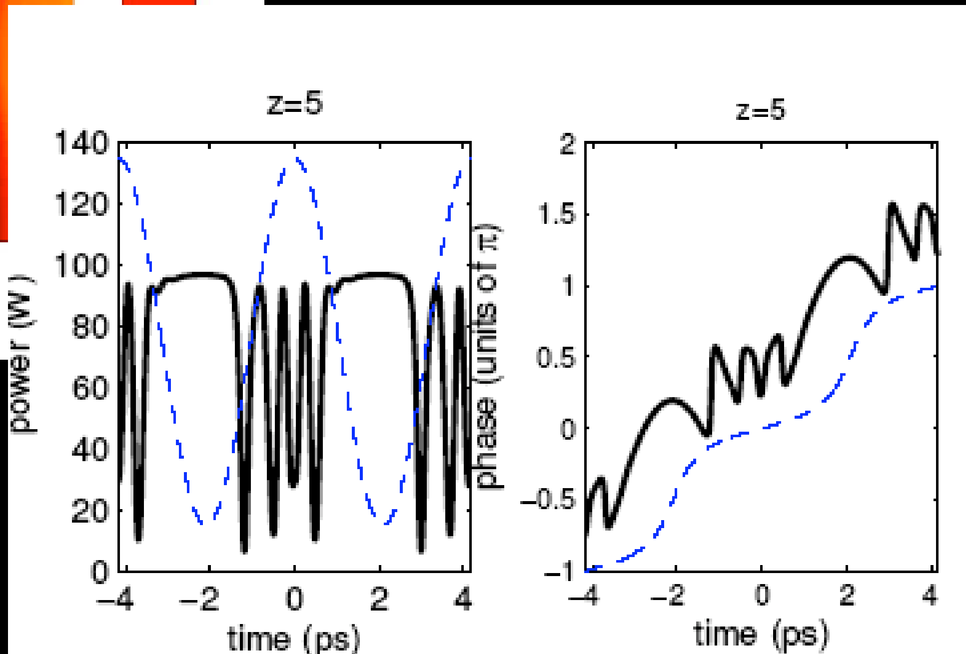
Experiment (temporal) vs theory - 1

L=5 m fiber; k''=64 ps^2/km ; γ=52 (Wkm)$^{-1}$; P_{tot}=15+60 W



ε=0.05

Δf=0.239 THz

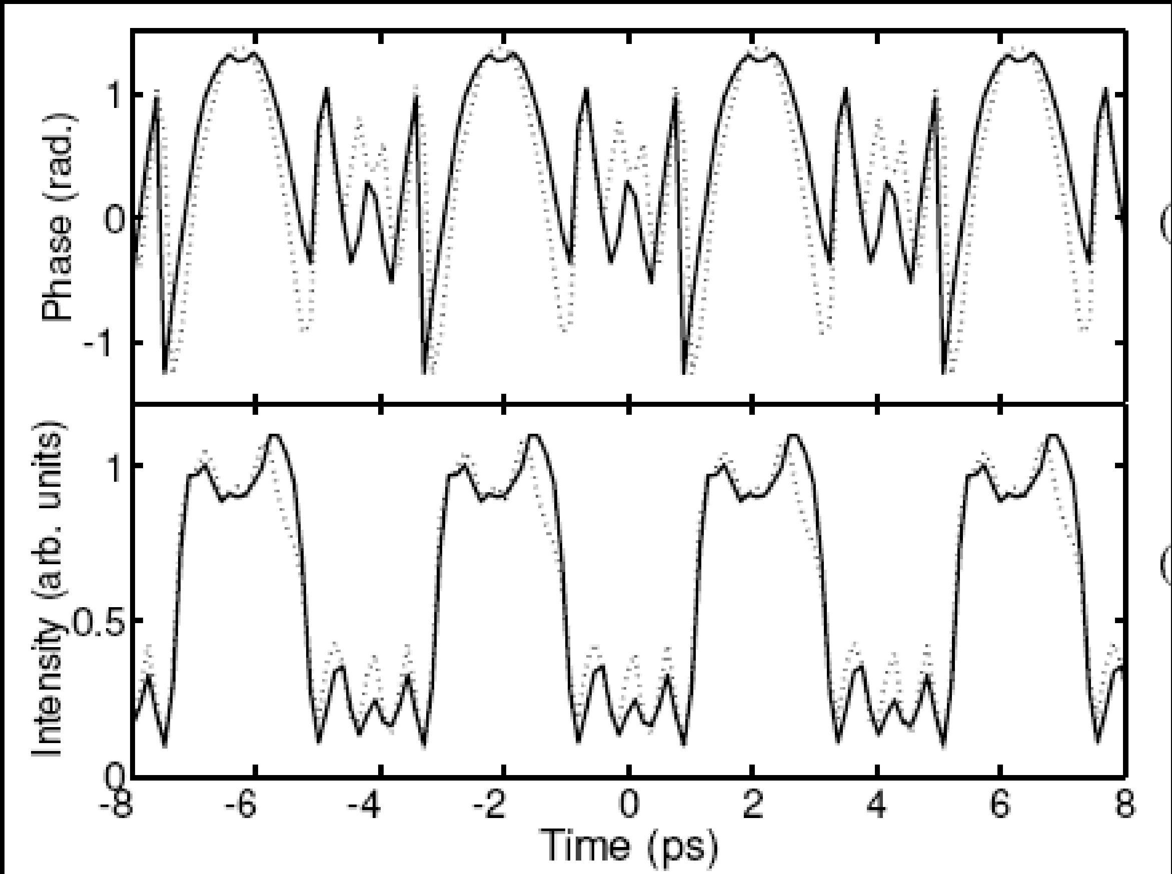


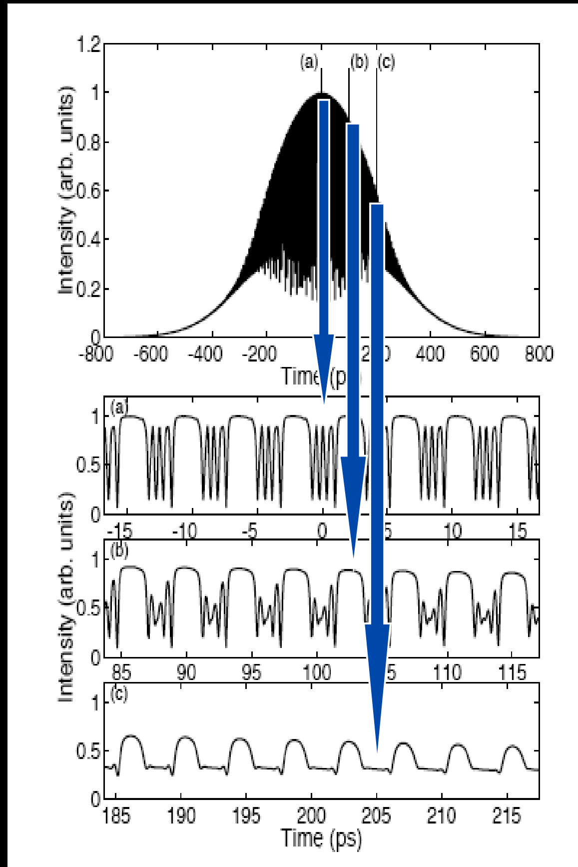
Experiment (temporal) vs theory - 2

Fiber output @ 5m: SHG-FROG retrieved data

exp (ps time scale) - dashed (theory)

calculated pulse 0.5 ns

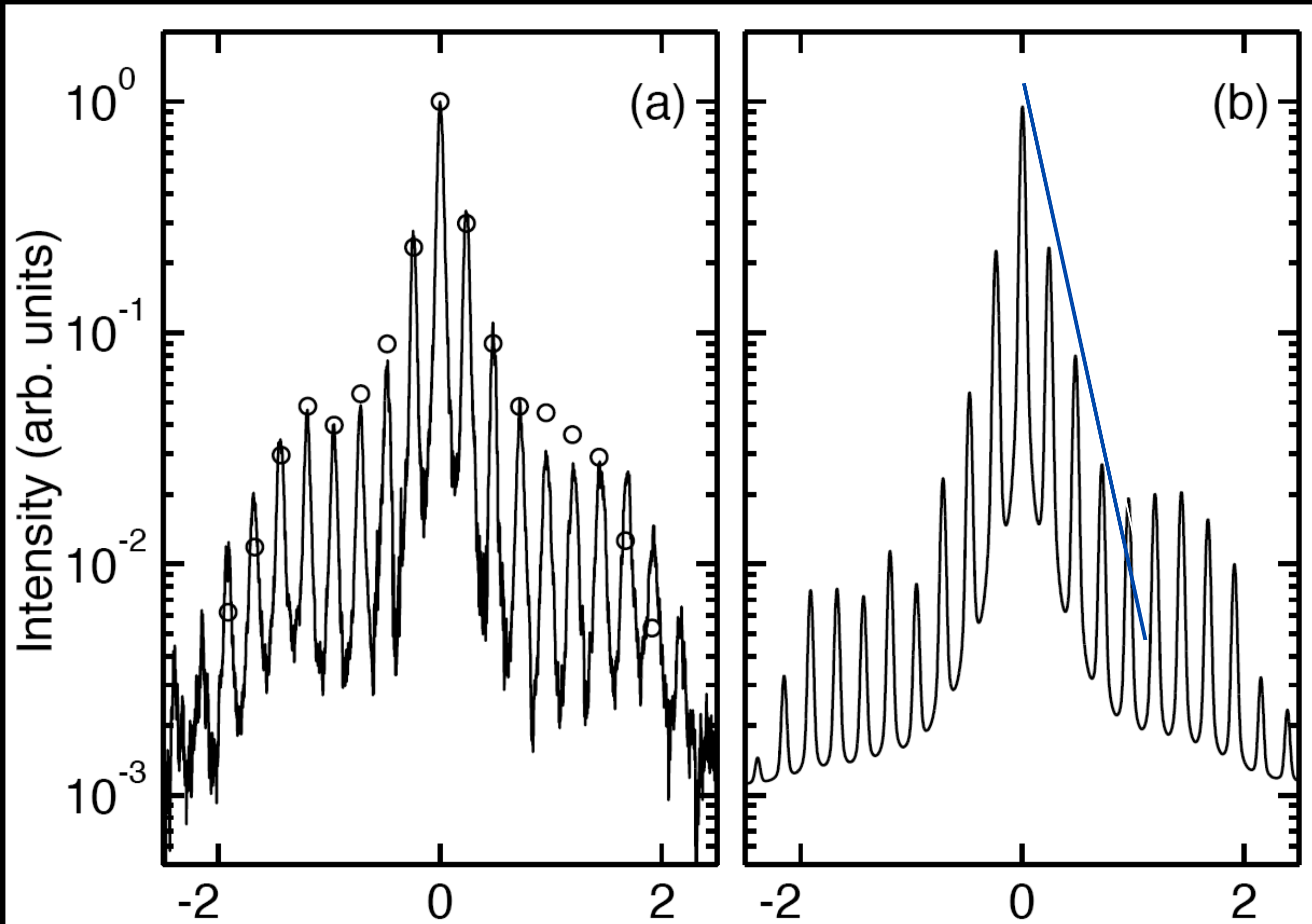




Experiment (temporal) vs theory - 3

Experiment
___ direct
o from FROG

calculated (NLS)



Frequency detuning (THz)

Conclusions

dispersive shock waves
NL optics can drive their general understanding

- shocks survive nonlocal (thermal) response
- occur for FWM (periodic) - multisoliton collisions