Efficient Solution of BVPs in ODEs and DAEs with Singularities

Ewa Weinmüller
(TU Wien Vienna University of Technology)

We deal with boundary value problems for systems of ordinary differential equations with singularities. Typically, such problems have the form

$$z'(t) = F(t, z(t)), \quad t \in (0, 1], \quad B_0 z(0) + B_1 z(1) = \beta$$

where $\lim_{t\to 0} F(t, z(t)) = \infty$ and $\lim_{t\to 0} \partial F(t, z)/\partial z = \infty$. The analysis is usually done for the model equation

$$z'(t) = \frac{1}{t^\alpha} M z(t) + f(t, z(t)), \quad t \in (0, 1], \quad B_0 z(0) + B_1 z(1) = \beta,$$

where $f(t, z)$ may also be in the form of $g(t, z)/t$ with a smooth function $g(t, z)$.

To compute the numerical approximation for z we use polynomial collocation, because the method retains its high order even in case of singularities. We will discuss how the collocation performs for problems with the inhomogeneity of the form $g(t, z)/t$.

The updated version of the MATLAB code `bvpsuite1.1` with the special focus on the above problem class has been implemented. Also systems of index 1 differential-algebraic equations (DAEs) are in the scope of the code. We illustrate the performance of the software with a special focus on parameter-dependent problems by means of numerical simulation of models in applications.