Exponential B-series and construction of sixth-order parallel exponential Runge–Kutta methods

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Exponential Runge-Kutta (ExpRK) methods are one of the two popular classes of one-step exponential integrators for solving stiff semilinear parabolic PDEs. It has been shown that the required number of stiff order conditions for ExpRK methods is the same as the one for the standard RK methods of the same order. This number is growing significantly as the order of the method increases, thereby causing difficulties in deriving order conditions and constructing schemes of high orders (which rely on solving a set of stiff order conditions involving matrix functions). Previous approaches were usually based on relaxing (weakening) many of the order conditions, resulting in schemes with reduced stability and must be implemented in a sequential way. Very recently, we were able to derive new ExpRK methods of orders 4 and 5 that require relaxing only one condition and have multiple independent stages. In this talk, with the help of the exponential B-series theory, we will derive 36 order conditions required for stiff ExpRK methods of order 6 and construct the first-ever sixth-order schemes without weakening any of the order conditions. While this requires a high number of stages, it is possible to construct multiple parallel stages which allow sixth-order schemes to behave like using with 6 stages and thus are efficient.