

***Nullspaces yield new explicit Runge–Kutta pairs*****James Verner****(Department of Mathematics, Simon Fraser University, Burnaby, BC)**

Sixty years ago, John Butcher developed a natural tabulation of order conditions for Runge–Kutta methods. Using certain 'simplifying conditions' to reduce the total number of order conditions, he solved these to provide some new explicit and implicit methods. A few years later, Fehlberg derived *pairs* of explicit methods of successive orders that could be implemented efficiently by using the difference of each pair of estimates to control the stepsize. Although Fehlberg's methods were deficient for quadrature problems, in the 1970's this author derived parametric families of explicit Runge–Kutta pairs of increasing orders 6 to 9 that avoided this problem altogether. Efficient examples have been selected from these families and implemented in both software packages and direct applications by a variety of authors and researchers.

Based on an algorithm taken from his Springer 2021 monograph, "B-Series: Algebraic Analysis of Numerical Methods", John has developed a MAPLE version 'Test 21' to solve the order conditions directly. As an exploration, this code was applied to derive 13-stage pairs of orders 7 and 8 from the parametric family characterized by the author. Unexpectedly, this execution revealed the existence of previously unknown pairs — i.e. formulas that satisfied some *but not all* of the previously assumed simplifying conditions. This talk reports formulas for directly deriving this new parametric family of (13,7-8) methods. This project was motivated by the possibility that similar formulas might be found to derive other new high-order Runge–Kutta or related methods.