A boundary-value-problem approach to phase sensitivity

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Attracting periodic orbits of vector fields are important objects of study, both from a theoretical and practical point of view. Especially when the underlying oscillation is far from sinusoidal— think pulsing lasers or spiking neurons — it is an interesting and challenging question to ask how the system relaxes back to the periodic orbit when it is subjected to a given perturbation. This question can be answered in a global and geometric way by considering the foliation of isochrons over the circle: each isochron comprises all points that converge to the periodic orbit with a given phase. Hence, isochrons are stabe manifolds of (fixed) points on the periodic orbit under the time-T map, where T is the period of the periodic orbit. In this talk, we show how isochron foliations, or the resetting behaviour related to a given perturbation, can be computed very accurately via a technique based on the continuation of solution families of a (multi-segment) boundary value problem. We apply this method to study the geometry of isochrons and their transformations in the context of increased phase sensitivity in the system, and utilise the approach to represent and illustrate two-dimensional global invariant manifolds in a new and efficient way.