Complex analysis applied to numerical differential equations

Taketomo Mitsui
(Nagoya University)

Complex function theory has several applications to numerical analysis of differential equations. Its typical example is stability analysis of discrete variable methods (DVMs) for ordinary differential equations (ODEs), because we are usually interested in the asymptotic solution behaviour of DVM applied to the scalar test equation $\mathrm{d}x/\mathrm{d}t = \lambda x$ ($\lambda \in \mathbb{C}, \mathrm{Re}\,\lambda < 0$). Then, we obtain a complex-variable function which governs the asymptotic behaviour of numerical solution with the argument variable $z = \lambda\times$(step-size). By the contour figure drawing we derive the stability region of Runge–Kutta methods, while an application of Schur algorithm leads to that of linear multistep methods. In contrast of this, the delay-differential equation (DDE) case gives the linear test equation $\mathrm{d}x/\mathrm{d}t\ (t) = Lx(t) + Mx(t-\tau)$ (L and M are constant matrices and τ is the delay) for asymptotic stability. We will explain an application of the argument principle of complex function to the linear stability analysis of DVM for DDEs.