Two geometric integrators

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This two-part talk will consider two geometric integrators, both based on (partitioned) Runge–Kutta.

The first part is a generalisation of the Kahan method, an RK method which preserves an invariant measure and energy for first-order quadratic Hamiltonian systems. The generalisation applies to higher order equations of higher degree, such as cubic second-order Hamiltonian systems. An example, introduced by Reinout Quispel and Andy Hone, is $x_2 - 2x_1 + x_0 = \epsilon x_0 x_1 x_2$. This approach is extended to ODEs of the form $x^{(n)} = P_{n+1}(x)$, $x \in R^m$, finding a first integral for $n = 2$, any m, and an invariant measure for any n and m. The methods used are a combination of Darboux polynomials and affine equivariance.

The second part concerns a class of methods, originally developed for use in celestial mechanics, formed by doubling the phase space to obtain a separable system, combined with a symmetric projection to the original phase space. I show that this in fact yields a mono-implicit symplectic Runge-Kutta method, prompting the consideration of this class of methods.