An Hermite–Obreshkov method for the Mathieu Equation

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The Mathieu differential equation

$$y'' + (a - 2q\cos 2x)y = 0$$

has among its solutions the so-called Mathieu functions, and if a complex variable is used, the so-called Modified Mathieu functions. Many software systems have code for computing these functions efficiently. However, in the case of double eigenvalues, which can occur for isolated complex values of the parameter q, more care is needed. To investigate this issue I wrote a special-purpose code in Maple. I briefly described this code in the paper "Computation and Applications of the Mathieu Functions, A Historical Perspective" with Chris Brimacombe and Mair Zamir, available open-access at `https://epubs.siam.org/doi/10.1137/20M135786X`, but the method has some interesting features and might be interesting for ANODE participants to consider: it is a variable-stepsize method of fixed but arbitrary order, as implemented. Indeed a variable order implementation might be even better. The Mathieu functions themselves are probably best computed by a spectral method, but the Modified Mathieu functions and other solutions may be more conveniently computed this way. I also did some numerical analysis of the Hermite interpolants, in a careful implementation, that might also be of interest. This is essentially a case study of a "boutique" numerical method.