High order Runge–Kutta methods revisited

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Sixth order methods were derived by Huťa [6] with eight stages and by Butcher [1] with seven stages. Eighth order methods, with 11 stages were derived by Curtis [3] and by Cooper and Verner [4].
A common pattern is proposed, for possible order $2n$ methods, based on the methods in [4] with $1 + \frac{1}{2}n(n+1)$ stages. For these methods, the abscissae are located at selections of the zeros of the shifted Lobatto polynomial of degree $n + 1$.
Methods with this design give optimal number of stages for orders 2, 4, 6 and 8.
This talk contains an approach to the analysis and derivation of these methods based on a sequence of diagonal blocks. The methods of [4] are derived in a new way but an attempt to find related methods of order 10 was unsuccessful. Hence, the famous method of Hairer [5], with 17 stages, seems to be optimal.
One of the tools used in this work has been a program written by Jim Verner and myself in January 1970 and recalled in [2]. I gratefully acknowledge this collaboration and the countless other ways we have shared ideas and supported each other over the years.

References

[1] J. C. Butcher, On Runge–Kutta processes of high order, J. Austral. Math. Soc., **4** (1964),179–194.

[2] J. C. Butcher and J. H; Verner, Our first B-series program, `https://jcbutcher.com/1970` (1970).

[3] A. R. Curtis, An eighth order Runge–Kutta process with eleven function evaluations per step, Numer. Math., **16** (1970) 268–277.

[4] G. J. Cooper and J. H. Verner, Some explicit Runge–Kutta methods of high order, SIAM J. Numer. Anal., **9** (1972) 389–405.

[5] E. Hairer, A Runge–Kutta method of order 10, J. Inst. Maths. Applics., **21** (1978), 47–59.

[6] A. Huťa, Une amélioration de la méthode de Runge–Kutta–Nyström pour la résolution numérique des équations différentielles du premier ordre. Acta Fac. Nat. Univ. Comenian. Math. **1** (1956) 201–224.